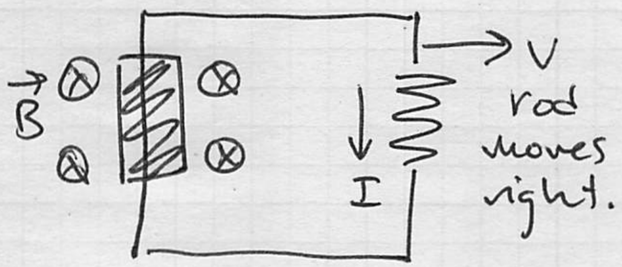
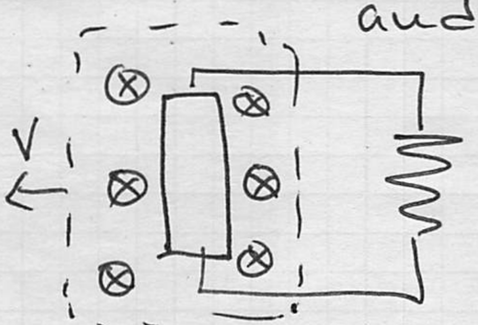
Here's two cases that are different, but related to eachother.

Case 1: canonical circuit moves out of B field. $\Rightarrow$ get a motional EMF



We know we get an EMF and that drives a current.

Case 2: the magnetic field is moved and the circuit remains fixed.



the region of magnetic field moves to the left and $V_{rod} = 0$ (it doesn't move)

Case 2 is different: $\vec{V} = 0$ such that $\vec{F} = q\vec{v} \times \vec{B} = 0$ there is no magnetic force on the charges.

But! Relativity suggests that w/ a simple frame shift there must be an EMF and thus a current must flow in Case 2.

Faraday conducted these experiments in the 1830s!

In case 1, we would say that

$\vec{f}$ is magnetic $\Rightarrow \mathcal{E} = \oint \vec{f} \cdot d\vec{l}$

and thus is EMF arises from $\vec{v} \times \vec{B}$ (it's a Motional EMF).

In case 2, $\mathcal{E}$ must take on the same value (if v is the same), but what is $\vec{f}$ in this case? $\vec{V}_{rod} = 0$ so it can't be a magnetic force. in the reference frame where the circuit is fixed!

$\Rightarrow$ Turns out that there is a $\vec{E}$-field in this frame!

The electric & magnetic field are not absolute quantities; they depend on the frame (Relativity is important here as we will see near the end of the course.)

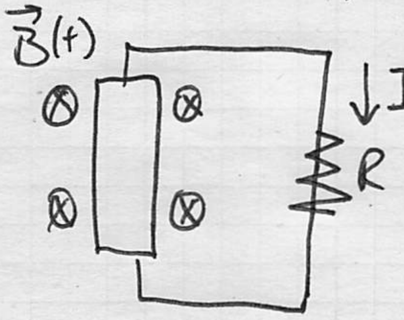
In either case, $\mathcal{E} = -d\Phi_B/dt$ works, which speaks to the utility of the concept of magnetic flux!

(in case 2, the moving magnetic field causes a change in the magnetic flux)

<u>Recall</u>: Lenz's Law helps us figure out the direction of the current. The EMF is generated to drive a current that <u>opposes</u> the change in flux!

Faraday also considered a third case,

Case 3: fixed (location) of $\vec{B}$ and circuit, but vary $\vec{B} = \vec{B}(t)$. in time.



For case 3, everything remains fixed in their locations, but the magnetic field varies in time $\Rightarrow$ current!

there $\mathcal{E} = -d\Phi_B/dt$ still works!

Faraday's experiments showed this.

$\Rightarrow$ Nothing is moving in any reference frame, so this absolutely <u>NOT</u> a motional EMF.

<u>Changing Magnetic Fields Drive Currents</u>

→ this is ~~one~~ a fact of nature; we observe that when B changes currents can be driven!

→ How does this happen? b/c only $\vec{E}$ can drive stationary charges.

Faraday postulated the a changing magnetic field would induce an electric field.

$$* \quad \boxed{\mathcal{E} = \oint \vec{E}_{NC} \cdot d\vec{l} = -\frac{d\phi_{mag}}{dt}}$$

Faraday's Law in Integral form.

* I use the subscript "NC" b/c this is not a coulombic $\mathcal{E}$ field. $\nabla \times E_{NC} \neq 0$ most of the time.

Faraday's Law - a Quick Derivation

We can construct the local statement of Faraday's Law using the global statement.

$$\oint \vec{E} \cdot d\vec{l} = \iint \nabla \times \vec{E} \cdot d\vec{A}$$

$$\Phi_{mag} = \iint \vec{B} \cdot d\vec{A}$$

Here I get rid of NC b/c we know $\vec{E}_{es}$ will have $\nabla \times \vec{E}_{es} = 0$.

$$\Rightarrow \frac{d\Phi_{mag}}{dt} = \frac{d}{dt} \iint \vec{B} \cdot d\vec{A} = \iint \frac{d\vec{B}}{dt} \cdot d\vec{A} \; \star$$

Here we consider $\mathcal{E}$ at an instant so that $\hat{n}$ + dA are not changing. This gives us.

$$\iint \nabla \times \vec{E} \cdot d\vec{A} = -\iint \frac{d\vec{B}}{dt} \cdot d\vec{A}$$

or

$$\iint \left(\nabla \times \vec{E} + \frac{d\vec{B}}{dt}\right) \cdot d\vec{A} = 0$$ for any surface S.

So that $\boxed{\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}}$ local statement of Faraday's Law.

True for every point in space.

The minus sign reminds us that the non-coulombic electric field will setup to oppose changes in magnetic flux (Lenz's Law)

This new $\vec{E}$ field is not a coulombic field $\Rightarrow$ not stemming from charges so,

$$\vec{E}_{tot} = \vec{E}_{es} + \vec{E}_{NC} \Rightarrow \nabla \times \vec{E}_{tot} = \nabla \times \vec{E}_{NC} \neq 0.$$

you can get curly E fields when $\vec{B} = \vec{B}(t)$.

When there are no source charges ($\rho=0$) then we have a set of equations that look quite similar,

local statements

$$\left\{\begin{array}{ll} \nabla\times\vec{E} = -\dfrac{d\vec{B}}{dt} & \text{Faraday's Law} \\ \nabla\times\vec{B} = \mu_0\vec{J} & \text{Ampere's Law} \\ \nabla\cdot\vec{E} = 0 & \\ \nabla\cdot\vec{B} = 0 & \end{array}\right.$$

very similar structure

when $\rho=0$ Faraday's Law problems can be solved like Ampere's problems.

In their integral form,

global statements

$$\left\{\begin{array}{l} \oint\vec{E}\cdot d\vec{l} = -d\Phi_B/dt \\ \oint\vec{B}\cdot d\vec{l} = \iint\mu_0\vec{J}\cdot d\vec{A} = \mu_0 I_{enc} \end{array}\right.$$

Because of the form Maxwell's equations take when $\rho = 0$, we can develop an analogy between Ampere's Law & Faraday's Law.

$$\nabla \cdot \vec{B} = 0 \longleftrightarrow \nabla \cdot \vec{E} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \longleftrightarrow \nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

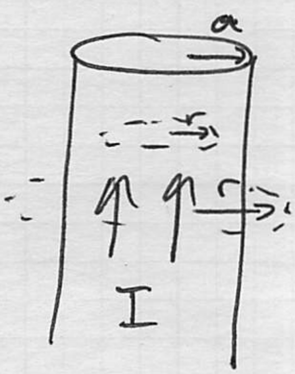
so that,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{A} = \mu_0 I_{enc} \longleftrightarrow \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

Recall that the contribution to this integral is only from non Coulombic sources.

Ampere's Example

Remember that we computed the magnetic field inside and outside of a thick wire.



outside: $r > a$ $\qquad \vec{B} = B(r)\hat{\varphi}$ by symmetry

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow B 2\pi r = \mu_0 I \qquad \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

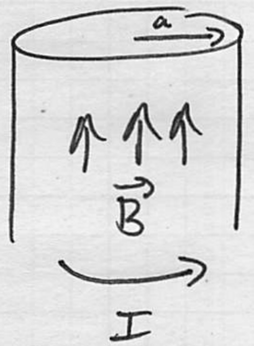
inside: $r < a$ $\qquad J$ is uniform: $J = \frac{I}{\pi a^2}$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow B 2\pi r = \mu_0 \iint \vec{J} \cdot d\vec{A} = \mu_0 J \pi r^2 = \frac{\mu_0 I \pi r^2}{\pi a^2}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi a^2} r \hat{\varphi}$$

We can use this analogy to determine the electric field around a solenoid,



If it has a n turns / length then $\vec{B} = \begin{cases} \mu_0 n I \hat{z} & \text{inside } r<a \\ 0 & \text{outside } r>a \end{cases}$

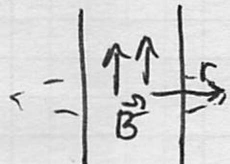
this magnetic field looks precisely like what we had for $\vec{J}$ ~~or~~ in the previous example.

If the current changes with time $(I = I(t))$, then so does the magnetic field $(\vec{B} = \vec{B}(t))$.

Farday's Law says,

$$\oint \vec{E} \cdot d\vec{l} = \frac{-d\Phi_{\text{thru loop}}}{dt}$$

We expect (as before) $\vec{E} = E(r)\hat{\varphi}$ so we can draw a Faraday Loop, $(r<a)$



$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = B\pi r^2$$
$$= \mu_0 n I \pi r^2$$

so that $\frac{d\Phi}{dt} = \mu_0 n \frac{dI}{dt} \pi r^2$

$\oint \vec{E} \cdot d\vec{l} = E 2\pi r$ so that,

$$\vec{E} = -\frac{1}{2\pi r} \mu_0 n \frac{dI}{dt} \pi r^2 \hat{\varphi}$$

$$\vec{E} = -\frac{\mu_0 n}{2} \frac{dI}{dt} r \hat{\varphi} \quad \text{inside}$$

$I \uparrow$ $\vec{E}$ goes ↻
$I \downarrow$ $\vec{E}$ goes ↺
Always "fight the change"

If you are outside the solenoid, we can use the same logic with $\Phi_{enclosed}$ stopping at $r=a$,

$$\oint \vec{E}\cdot d\vec{l} = -d\phi/dt$$

ϕ is zero outside the solenoid b/c $B=0$.

$$E\,2\pi r = -\frac{d}{dt}\left(\mu_0 n I \pi a^2\right)$$

$$\vec{E} = -\mu_0 n \frac{dI}{dt}\frac{\pi a^2}{2\pi r}\hat{\varphi} = -\mu_0 n \frac{dI}{dt}\frac{a^2}{2r}\hat{\varphi}$$

same direction as the previous result.

But notice! $\vec{B}=0$ out there and yet $\vec{E}$ exists throughout space.

this is interesting. we can have a localized source and yet generate something that lives throughout space. (Similar to $\rho \to \vec{E}$ & $\vec{J} \to \vec{B}$)

<u>Comments on the Curl</u>

outside the field is $\vec{E} \propto \frac{1}{r}\hat{\varphi}$.

this field circles around, but it has no curl!

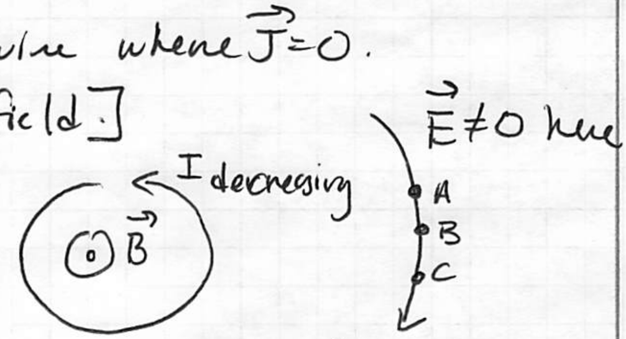
??? $\nabla\times\vec{E} = -d\vec{B}/dt = 0$ out here!

Just like $\vec{B} \propto \frac{1}{r}\hat{\varphi}$ outside a wire where $\vec{J}=0$.

[$1/r\,\hat{\varphi}$ is a very special field]

But here's a problem,

with this $V_A > V_B > V_C$ →



so that $V_A > V_A$ if we go around!

V is no longer well defined it's path dependent. $\nabla\times\vec{E}\neq 0$ everywhere.

This is why we talk about EMF + not ΔV

$$\oint \vec{E}\cdot d\vec{l} = 0 \quad \text{so that} \longrightarrow \int_A^B \vec{E}\cdot d\vec{l} = V_A - V_B$$

if the path contains no changing flux

is well defined

But if our path contains changing flux then,

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt} = \mathcal{E}MF$$

We can still compute $\int_A^B \vec{E}\cdot d\vec{l}$ if the path is defined, but this calculation is not path independent.

Voltage/Potential lose some of their meaning.