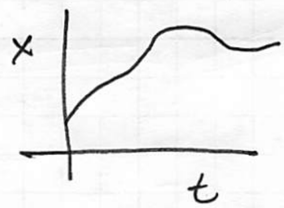
We've done alot with equations including developing a matrix formalism for the Lorentz transformations $x'^{\mu} = \Lambda^{\mu}_{\ \nu} x^{\nu}$

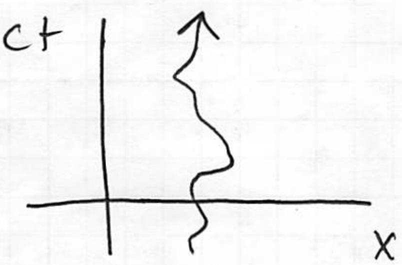
Our usual graphical representation of 1D motion is



where position is on the y-axis (vertical) and time is on the "x"-axis (horizontal)

this gives us: $V_x = \frac{dx}{dt} \approx \frac{\Delta x}{\Delta t}$ and $a_x = \frac{d^2x}{dt^2} \approx \frac{\Delta V_x}{\Delta t}$

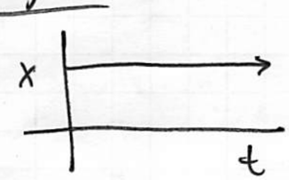
In special relativity, we use space time diagrams or "Minkowski" diagrams, which flip the axis and also use the 4-vector components as the coordinate axis. In "1D" this consists of $x^0 = ct$ and $x^1 = x$,



This is a "world line", the trajectory of an object through space & time.

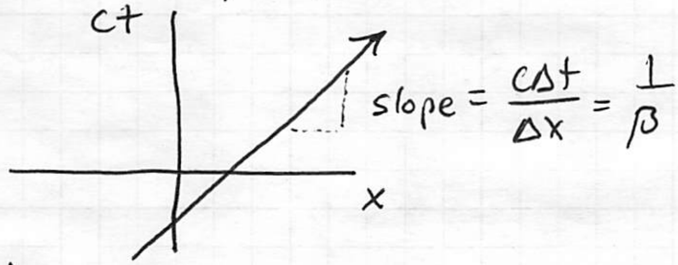
An object at rest would look like this

Regular



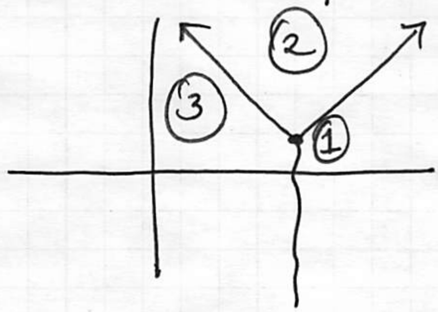
Minkowski

On these graphs there can be no slope less than 1, $\beta \leq 1$ $(\beta = v/c)$

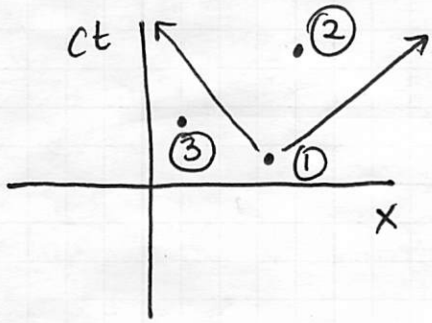


When the slope = 1, we refer to those lines as the "light cones." (If you add y they are cones and with z, hypercones)

A particle is only able to communicate inside its forward light cone,



Event 2 could be caused by event 1, but event 3 could <u>not</u> be caused by event 1.

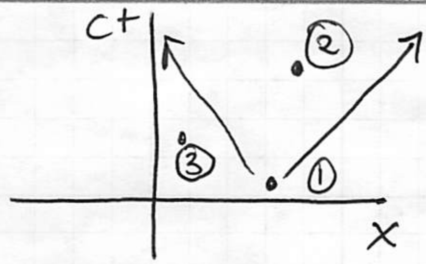


$I_{21} < 0$ here, events 2 and 1 are timelike separated. $\Delta x_{12}^2 < c^2 \Delta t_{12}^2$ if 2 is in the light cone of 1.

Thus there's no frame where $\Delta t_{12} = 0$.

Where $\Delta x_{12} = 0$, Δt_{12} is at a minimum, (because its the proper time, the events happen at the same location)

You can never <u>reverse</u> the time order of the events. Cause precedes effect in all frames.



$I_{13} > 0$ here, Events 1 and 3 are space like separated

$\Delta x_{13}^2 > c^2 \Delta t_{13}^2$

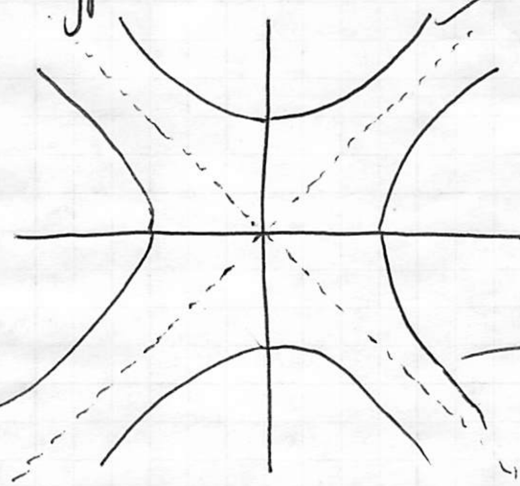
Here you can reverse the time order as Δt_{13} can be positive, negative, or zero.

But thats totally OK because 1 can't cause 3. There's no violation of causality → they are just "elsewhere".

Light signals are the fastest possible "causes" so that $\Delta t = \frac{\Delta x}{c}$ and Δx_{13} is larger than that.

Events with common space-time intervals

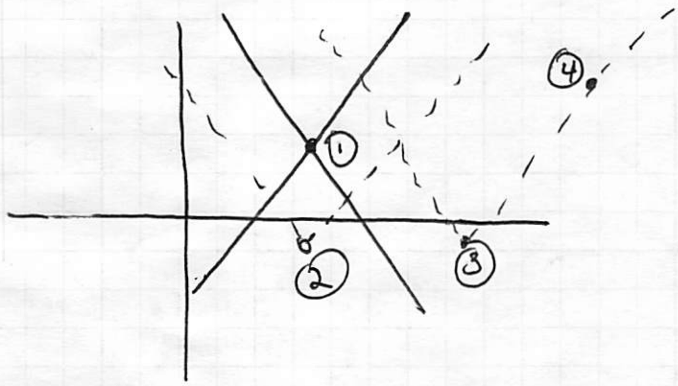
Earlier, we defined $I \equiv \Delta x^2 - c^2 t^2$ to be a space-time interval. All the events with the same space time interval exist on a hyperbole.



→ These events all have a common I with respect to the origin. (they are all time like)

→ These events also have a common I but they are space like..

Lorentz boosts can only move you to a diffrent point on the same hyperbole (as they keep I constant) → you cannot boost from a forward light cone to a backwards one.



Event 2 can be causally related to 1, but not 3.

4 is on the light cone of 3.