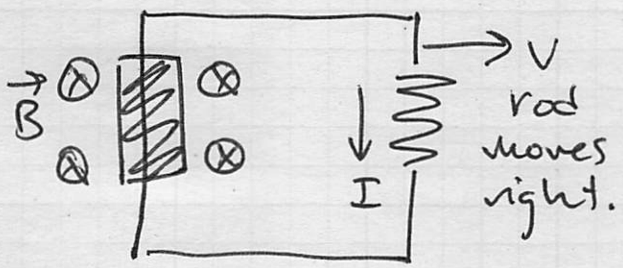
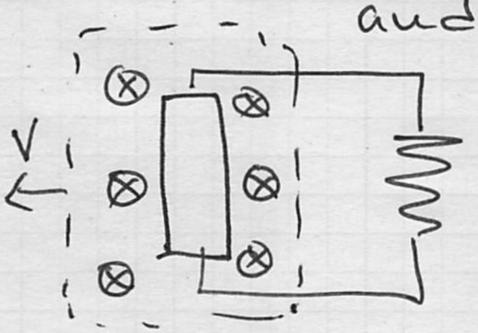
Here's two cases that are diffrent, but related to eachother.

<u>Case 1:</u> canonical circuit moves out of B field. $\Rightarrow$ get a motional EMF




we know we get an EMF and that drives a current.

<u>Case 2</u>: the magnetic field is moved and the circuit remains fixed.




the region of magnetic field moves to the left and $V_{rod} = 0$ (it doesn't move)

Case 2 is different: $\vec{V} = 0$ such that $\vec{F} = q\vec{v} \times \vec{B} = 0$ there is no Magnetic force on the charges.

<u>But!</u> Relativity suggests that w/ a simple frame shift there must be an EMF and thus a current must flow in Case 2.

Faraday conducted these experiments in the 1830s!

In Case 1, we would say that
$\vec{f}$ is magnetic $\Rightarrow$ $\mathcal{E} = \oint \vec{f}\cdot d\vec{l}$
and thus is EMF arises from $\vec{v} \times \vec{B}$
(it's a Motional EMF).

In case 2, $\mathcal{E}$ must take on the same value
(if v is the same), but what is $\vec{f}$ in this
case? $\vec{V}_{rod} = 0$ so it can't be a magnetic
force. in the reference frame where the
circuit is fixed!

$\Rightarrow$ Turns out that there is a $\vec{E}$-field
in this frame!

The electric & magnetic field are not
absolute quantities; they depend on
the frame (Relativity is important here
as we will see near the end of the course.)

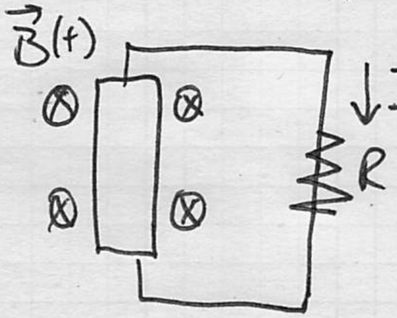
In either case, $\mathcal{E} = -d\Phi_B/dt$ works, which
speaks to the utility of the concept
of magnetic flux!

(in case 2, the moving magnetic field causes
a change in the magnetic flux)

<u>Recall</u>: Lenz's Law helps us figure out
the direction of the current. The EMF
is generated to drive a current that
<u>opposes</u> the change in flux!

Faraday also considered a third case,

Case 3: fixed (location) of $\vec{B}$ and circuit, but vary $\vec{B} = \vec{B}(t)$. in time.




For case 3, everything remains fixed in their locations, but the magnetic field varies in time $\Rightarrow$ current!

there $\mathcal{E} = -d\Phi_B/dt$ still works!

Faraday's experiments showed this.

$\Rightarrow$ Nothing is moving in any reference frame, so this absolutely NOT a motional EMF.

## Changing Magnetic Fields Drive Currents

$\rightarrow$ this is ~~one~~ a fact of nature; we observe that when B changes currents can be driven!

$\rightarrow$ How does this happen? b/c only $\vec{E}$ can drive stationary charges.

Faraday postulated the a changing magnetic field would induce an electric field.

$$* \boxed{\mathcal{E} = \oint \vec{E}_{NC} \cdot d\vec{l} = -\frac{d\phi_{mag}}{dt}}$$

Faraday's Law in Integral form.

* I use the subscript "NC" b/c this is not a coulombic $\mathcal{E}$ field. $\nabla \times E_{NC} \neq 0$ most of the time.

## Faraday's Law - a Quick Derivation

We can construct the local statement of Faraday's Law using the global statement.

$$\oint \vec{E}\cdot d\vec{l} = \iint \nabla\times\vec{E}\cdot d\vec{A}$$

Here I get rid of NC b/c we know $\vec{E}_{es}$ will have $\nabla\times\vec{E}_{es}=0$.

$$\Phi_{mag} = \iint \vec{B}\cdot d\vec{A}$$

$$\Rightarrow \frac{d\Phi_{mag}}{dt} = \frac{d}{dt}\iint \vec{B}\cdot d\vec{A} = \iint \frac{d\vec{B}}{dt}\cdot d\vec{A} \; \star$$

Here we consider $\mathcal{E}$ at an instant so that $\hat{n}$ + $dA$ are not changing. This gives us.

$$\iint \nabla\times\vec{E}\cdot d\vec{A} = -\iint \frac{d\vec{B}}{dt}\cdot d\vec{A}$$

or

$$\iint \left(\nabla\times\vec{E} + \frac{d\vec{B}}{dt}\right)\cdot d\vec{A} = 0$$ for any surface S.

So that $\boxed{\nabla\times\vec{E} = -\frac{d\vec{B}}{dt}}$ local statement of faraday's Law.

True for every point in space.

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The minus sign reminds us that the non-coulombic electric field will setup to oppose changes in magnetic flux (Lenz's Law)

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This new $\vec{E}$ field is not a coulombic field → not stemming from charges so,

$$\vec{E}_{tot} = \vec{E}_{es} + \vec{E}_{NC} \Rightarrow \nabla\times\vec{E}_{tot} = \nabla\times\vec{E}_{NC} \neq 0.$$

you can get curly E fields when $\vec{B}=\vec{B}(t)$.

When there are no source charges ($\rho=0$) then we have a set of equations that look quite similar,

local statements:

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \quad \text{Faraday's Law}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \text{Ampere's Law}$$

very similar structure

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

when $\rho = 0$ Faraday's Law problems can be solved like Ampere's problems.

In their integral form,

global statements:

$$\oint \vec{E} \cdot d\vec{l} = -d\Phi_B/dt$$

$$\oint \vec{B} \cdot d\vec{l} = \iint \mu_0 \vec{J} \cdot d\vec{A} = \mu_0 I_{enc}$$