Up to now, we've been looking at pieces or chunks of wire (resistors), so let's out and look at complete circuits to see how we under current flow in them.* Here, we will expand on 184 material

- If current flows around a circuit, it must be driven in some way → we've described the $\vec{E}$ field insite the wire/resistors.
- A measure of this driving is called EMF (Historically, the electromotive force, which is a terrible name as it's not a force!)

EMF

$$\mathcal{E} \equiv \text{EMF} \equiv \oint_{\substack{\text{a circuit}\\ \text{loop}}} \vec{f} \cdot d\vec{l}$$

[gives this measure at one instant in time when $\vec{f}$ is known]

$\vec{f}$ is the usual Force/charge.

The definition of EMF looks kind of like a ~~force~~ work/unit charge taken once around the loop.

$\vec{f}$ here can be any force that drives charges:

Electric $\vec{f} = \vec{E}$

Magnetic $\vec{f} = \vec{v} \times \vec{B}$

physical pushes (like "ants") $\vec{f} = \vec{F}_{ant}/q$

Sometimes it's the case that $\vec{f}$ is hard to define for us (like in a battery where chemistry & QM are needed to understand the physics of $\vec{f}$)

Units of EMF : $[\text{EMF}] = \left[\frac{\text{Work}}{\text{Charge}}\right] = \text{J/C} = \text{Volt}.$

Where does the EMF come from?

If your circuit involved only electrostatic forces then

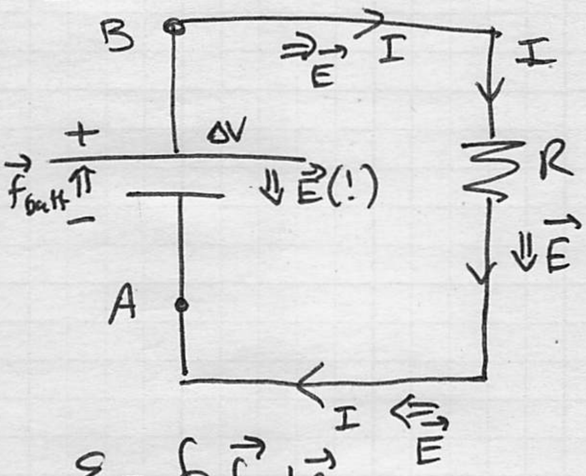
$$\mathcal{E}_{\substack{\text{pure}\\ \text{e-static}}} = \oint \vec{E}_{\substack{\text{electro}\\ \text{static}}} \cdot d\vec{l} = 0$$

Because $\nabla \times \vec{E}_{\text{electrostatic}} = 0$

So to drive a current around a circuit, you need another force!

In a battery → it's chemical
In VandeGraaff → it's mechanical
* In a generator → it's magnetic ($\vec{v} \times \vec{B}$, "motional EMF")
↳(We will study this in some detail soon.)

A simple Circuit



In this simple cirwit, the Battery drives a current I thurgh the wires / resistor R.

The field in the wires are due to static charges on the surface of the wires, so this $\vec{E}$ is electrostatic.

$$\mathcal{E} = \oint \vec{f} \cdot d\vec{l}$$

So the only "non-electrostatic" force is inside the battery, $\vec{f}_{batt}$. In the battery, charges are driven from − to + by chemical force we are not investigating.

If the current is steady AND there's no resistance in the battery (Assume: Ideal Battery) then

* $\vec{f}_{batt} = -\vec{E}$ so that $\vec{f}_{net}$ in the battery is 0, no charge acceleration they have a steady flow.

* Note how this model differs from the Drude Model in the wires/resistor (collisions) with the same result → steady current.

- Everywhere else in the circuit the field/force is electrostatic. $\vec{f} = \vec{E}_{electrostatic}$.

So the EMF can be computed thusly,

$$\mathcal{E} = \oint \vec{f}_{total} \cdot d\vec{l} = \underset{\substack{\text{though}\\\text{battery}}}{\int_A^B} \underset{\substack{\uparrow\\\text{chemisty}}}{(\vec{f}_{batt} + \vec{E})} \cdot d\vec{l} + \underset{\substack{\text{rest of}\\\text{circuit}}}{\int_B^A} \underset{\substack{\uparrow\\\text{no chemisty}}}{\vec{E} \cdot d\vec{l}}$$

$$\mathcal{E} = \int_A^B \vec{f}_{batt} \cdot d\vec{l} + \oint \vec{E} \cdot d\vec{l} \Big\} \text{ this term is zero b/c } \nabla \times \vec{E} = 0$$

So the EMF is determined by the battery force.

$$\mathcal{E} = \int_A^B \vec{f}_{batt} \cdot d\vec{l}$$

We can manipulate the expression differently to show $\mathcal{E} = \Delta V_{batt}$.

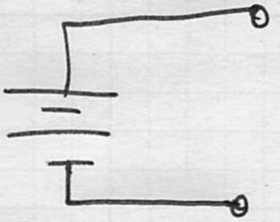
$$\mathcal{E} = \underset{\text{in batt}}{\int_A^B} \underbrace{(\vec{f}_{batt} + \vec{E})}_{0 \text{ b/c } \vec{f}_{batt} = -\vec{E}} \cdot d\vec{l} + \underset{\text{Outside}}{\int_B^A} \vec{E} \cdot d\vec{l}$$

$$= \int_B^A \vec{E} \cdot d\vec{l} = -\int_A^B \vec{E}\, d\vec{l} = \Delta V_{batt}!$$

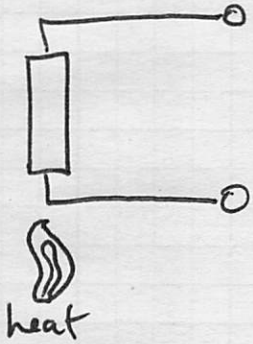
So the EMF is derived from the force delivered to charges by the battery and is equal to the potential difference between the terminals of the battery (in the case of an ideal battery).

There are a number of common sources of EMF

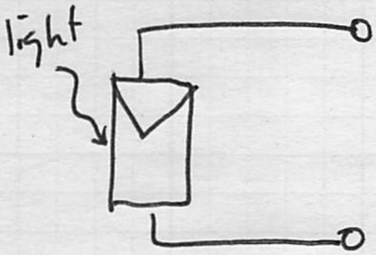
- Anything that converts energy input into the ability to do work.



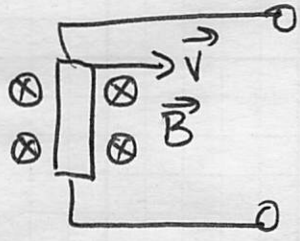
A battery that through chemical reactions creates a potential difference



A thermoelectric device that converts thermal energy into electrical energy



A photovoltaic device that converts light (photons) into electrical energy



A motional EMF that is generated by moving charges through B-fields*

*And in other ways, too.

All of these methods could do work!