So far we have found one way to determine the field in and around a material.

$$\text{Find } \vec{J}_B + \vec{K}_B \text{ from } \vec{M} \Rightarrow \text{compute } \vec{A} \text{ using integral} \Rightarrow \text{compute } \vec{B} \text{ using } \nabla\times\vec{A}.$$

- But we can be a bit more clever, especially if we think about how these currents and any free currents show up in our PDEs that describe the magnetic field. Let's see how.

- In general, any material could contain free currents (essentially, wires embedded in the material, free flowing ions, etc.) and, as a result, $\vec{B}$ fields appear which further magnetize the material, altering the field even more! How do we deal with all this?

Let's consider a total current density that is made up of these free currents and bound currents.

$$\vec{J} = \vec{J}_{free} + \vec{J}_{Bound}$$

This is the real current density that creates the real $\vec{B}$ field, $\oint \vec{B}\cdot d\vec{l} = \int \mu_0 \vec{J}\cdot d\vec{a}$, via Ampere's Law

$\vec{J}_{free}$: current that you control; you inject this

$\vec{J}_{Bound}$: how the material responds; inherent to material

Ampere's Law is always true in magnetostatics.

$$\nabla\times\vec{B} = \mu_0\vec{J} = \mu_0\left(\vec{J}_{free} + \vec{J}_{Bound}\right)$$

$$= \mu_0\left(\vec{J}_{free} + \nabla\times\vec{M}\right)$$

So that

$$\nabla \times \left(\vec{B} - \mu_0 \vec{M} \right) = \mu_0 \vec{J}_{free} \text{ or,}$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_{free}$$

We define a new field called "$\vec{H}$" that is mathematically equal to the quantity in paranthreses,

$$\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M} \quad \text{such that,}$$

$$\nabla \times \vec{H} = \vec{J}_{free} \Rightarrow \int \nabla \times \vec{H} \cdot d\vec{a} = \int \vec{J} \cdot d\vec{a}$$

or $$\oint \vec{H} \cdot d\vec{l} = \int \vec{J}_{free} \cdot d\vec{a} = I_{free,\ enclosed} \,!$$

[this is often very easy to measure as it is usually just the current in the wires that you control]

- the units of H are Amps/meter not tesla
- we just call this the H-field no real special name.

Note! this is a very similar story to what we found for the electric field in matter,

Gauss for $\vec{D}$: $$\oint \vec{D} \cdot d\vec{a}' = \int \rho_{free} \, d\tau' = Q_{free,\ enclosed}$$

where $$\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}$$

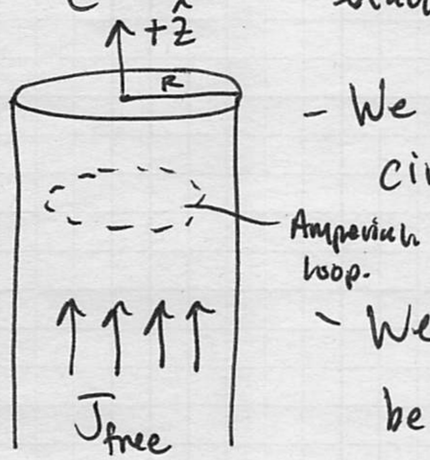
What we have found above is Ampere's Law for $\vec{H}$ and its relationship to <u>free</u> currents.

Example: Aluminum rod with uniform free current

Consider a long Al rod with radius R that carries a uniform free current J_{free} (total current $I = J_f \pi R^2$) in the $+z$ direction.

Let's try to find $\vec{B}$ & $\vec{H}$ everywhere.

Note: this is like Ex. 6.2 in Griffiths with Copper, which is a dia magnet, but Al is a para magnet, so it's worth comparing this example with his.



- We should expect that $\vec{B}$ will be circumferential $\vec{B}$ (just like usual w/ this kind of current)
- We also expect $\vec{M}_{inside}$ to be parallel to $\vec{B}$ because Al is a paramagnet. $\vec{M}$

Outside in space is vacuum so that $\vec{M}_{outside} = 0$.

With $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$ these two contributions $\vec{B}$ & $\vec{M}$ are subtracted.

- But, $\vec{M}$ is weak for most materials, so we can be pretty sure that $\vec{H}$ is still parallel to $\vec{B}$ & $\vec{M}$ $\vec{H}$.

If you aren't convinced think about the Amperian loop above and Ampere's for $\vec{H}$,

$$\oint \vec{H} \cdot d\vec{l} = I_{free,\,enclosed}$$

$\vec{H}$ looks like it points in $\vec{B}$'s direction.

We can use Ampere's Law for $\vec{H}$ inside and out,

s<R:

$$\oint \vec{H}\cdot d\vec{l} = I_{free,\,enclosed} \Rightarrow H 2\pi s = J_f \pi s^2$$

$$\vec{H} = \frac{J_f s}{2}\hat{\varphi} = \frac{I}{2\pi R^2} s\hat{\varphi} \qquad \text{with } I = J_f \pi R^2$$

[This is in fact the same result from Griffiths. Diamagnetic or paramagnetic, it makes no difference to $\vec{H}$ → only cares about <u>free currents</u>.]

s>R:

$$\oint \vec{H}\cdot d\vec{l} = I_{free} = I \Rightarrow H 2\pi s = I$$

$$\vec{H} = \frac{I}{2\pi s}\hat{\varphi}$$ [Again same result for diamagnetic]

<u>Can we now find the magnetic field $\vec{B}$?</u>

<u>Outside?</u> $\vec{M} = 0$ so $\vec{H} = \vec{B}/\mu_0$ thus,

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi s}\hat{\varphi}$$ (usual infinite wire result)

<u>Inside?</u>

Well we know the direction of $\vec{M}$ and we expect it to be less than $\vec{B}/\mu_0$ as $\vec{H}$ is still parallel to $\vec{B}$.

⇒ But we're stuck w/o knowing how Aluminium magnetizes (precisely)

In principle we can argue what we expect $\vec{M}$ to look like and thus what $\vec{K}_B + \vec{J}_B$ will look like, but w/o more information we can't compute either of them!