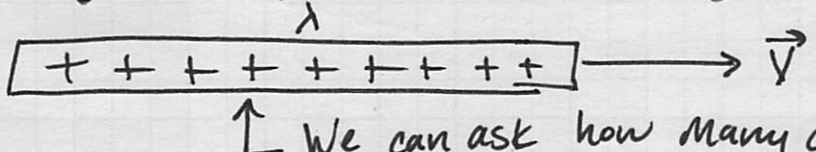
Current is the source of magnetic fields (and it also reacts in magnetic fields), so its very important to define it clearly and understand it. Current is a measure of the flow rate of charge, that is, "how many charges pass by each second."

$$|I| = dQ/dt$$ is how we defined current in Phy 184

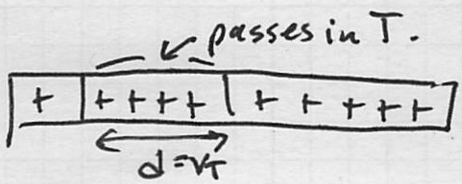
We need a more sophisticated understanding of current now and thus a more sophiscated and complete definition of current.

Consider a line charge with λ Coulombs/meter moving steadily with a velocity $\vec{v}$



We can ask how many charges (coulombs) move past this point in a time T.

Because $vT = d$ all the charges in a length d will pass that point in a time, T.



The total charge passing that location is Q,

$$Q = \lambda d = \lambda v T$$

So the rate that charge passes that location is,

$$\frac{Q}{T} = \frac{\lambda v T}{T} = \lambda v$$

So we can define current as follows,

$$\vec{I} = \lambda \vec{v}$$

Griffiths defines current as a vector and so will we, but not everyone does so.

Note: If λ is negative, the charges move in the other direction so you get the same current

$\xleftarrow{v_-} \ominus$ is the same as $\oplus \xrightarrow{v_+}$

Both have $I \rightarrow$ CO! ions

A few notes about current, I

- Current is measured in $\frac{\text{coulomb}}{\text{second}}$ = Amperes

  $1\,{}^{C}/_{c} = 1A.$

- If a wire has a known number of charge carriers per unit length, $n_L = \frac{\text{charge carriers}}{\text{length}}$, each with a charge, q, then,

  $$\lambda = n_L q \qquad \frac{\text{coulomb}}{\text{meter}} = \frac{\text{carriers}}{\text{meter}} \cdot \frac{\text{coulomb}}{\text{carrier}}$$

  so, $\vec{I} = n_L q \vec{v}$

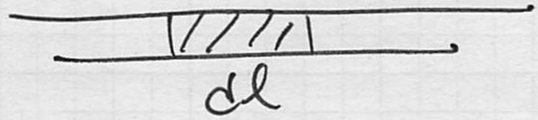
Forces on current carrying wires

The Lorentz force, rather the magnetic piece of that force, on a given charge is,

$$\vec{F} = q \vec{v} \times \vec{B}$$

Each charge moving in a wire feels a force in a magnetic field, so the "current" will feel the sum of those individual forces.

Consider a small piece of wire,



that has n_L charge carriers/length.

In each piece of length, dl, there are $n_L dl$ charges in the wire.

Each charge experiences a force,

$$\vec{F} = q\vec{v} \times \vec{B}$$

So in a chunk, dl, we find the force to be the superposition of all the forces on all the charges,

$$d\vec{F}_{\text{on chunk}} = n_L(dl)\, q\vec{v} \times \vec{B}$$

$\vec{v}$ is along the wire and so is $d\vec{l}$, so

$$\vec{v}\,dl = v\,d\vec{l}$$

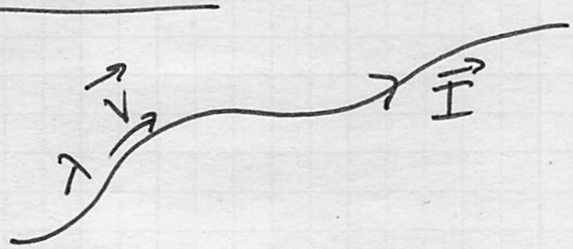
We can rewrite the force on the chunk,

$$d\vec{F} = n_L q v\, d\vec{l} \times \vec{B} = I d\vec{l} \times \vec{B}$$

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

I is the magnitude of the current + $d\vec{l}$ gives the direction

To summarize:



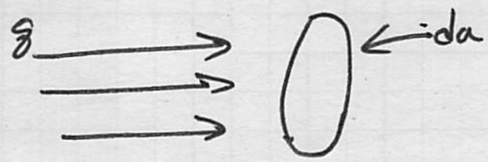
$$\vec{I} = \lambda\vec{v} = n_L q \vec{v}$$

force on wire chunk: $d\vec{F} = I d\vec{l} \times \vec{B}$

Charges don't just move in a line, that is, we cannot always model our current as an infinitesimally thin wire. Sometimes, the current is distributed over some volume or area that we need to understand.

Volume Charge Density, $\vec{J}$

The canonical setup for this is some charges moving throughout a volume, which cross some area, da.



We can define the current that passes through our little area da as usual,

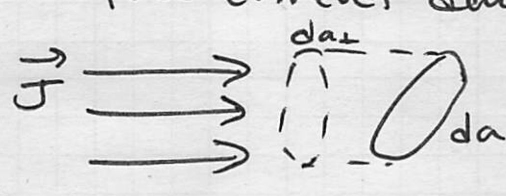
$$\text{current} = \frac{\text{total charge}}{\text{second}} = dq/dt$$

If da is very small, then the current, dI, that crosses the area is uniform so we can define a current density,

$$\vec{J} \equiv \begin{matrix}\text{volume}\\ \text{current}\\ \text{density}\end{matrix} = \frac{d\vec{I}}{da_\perp}$$

$\vec{J}$ and $d\vec{I}$ have the same direction.

da here needs to be the area perpendicular to the current density



so, $d\vec{I} = \vec{J}\, da_\perp = \vec{J} \cdot d\vec{a}$

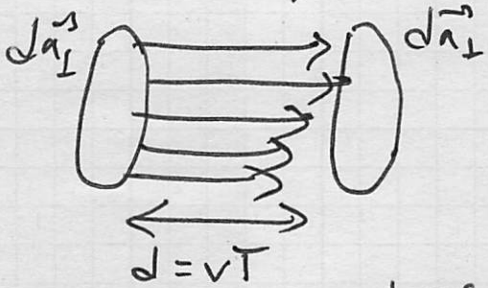
This expression also uniquely defines $\vec{J}$

(Q: what's J?

Just like with the line charge we can imagine how much charge crosses $da_\perp$ in a time, T.

It will be however much charge is in a distance $d = vT$, so,

$$Q = \rho v T \qquad \text{where } \rho = \text{charge density} = \frac{Q}{V}$$



So
$$d\vec{I} = \frac{Q}{T} = \rho \vec{v}\, da_\perp = J\, d\vec{a}_\perp$$

Thus the volume current density is related to the charge density,

$$\vec{J} = \rho \vec{v} \qquad \text{units? } \left[\frac{A}{m^2}\right]$$

<u>What about the force on this distribution?</u>

If this distribution is placed in an external Magnetic field, $\vec{B}$, how does the force on a chunk of the distribution relate to $\vec{B}$?

If N is the $\frac{\text{number of charge carriers}}{\text{volume}}$ then,

$\rho = Nq$ where q is the charge of the carrier itself.

$\vec{J} = Nq\vec{v}$ so we just add up all the forces on charges!

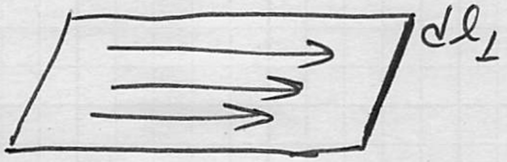
$$d\vec{F} = N\,d\tau\, q\,\vec{v} \times \vec{B} \quad \left(\begin{matrix}\text{total charge in } d\tau\\ \text{is } N d\tau!\end{matrix}\right)$$

so

$$d\vec{F} = (\vec{J} \times \vec{B})\, d\tau$$

We dealt with line currents, and volume currents, but these currents can exist on surfaces, too.

Surface Current density, $\vec{K}$



Here we track the number of charges passing the line segment $dl_\perp$

As usual, $dI = dq/dt$.

We will define a surface current density,

$\vec{K} \equiv \frac{d\vec{I}}{dl_\perp}$ (the direction of $\vec{K}$ is the direction of $d\vec{I}$) CQ: what's $\vec{K}$?

This idea is a bit difficult compared to the previous two. It's a ribbon of current and K tells us how much current passes by a unit length perpendicular to the flow.

We can quickly derive the relationship with surface current and the force on a surface current as we did before,

$\vec{K} = \sigma\vec{v}$ with σ = surface charge density $= \frac{C}{m^2}$

$= n_s q \vec{v}$ with $n_s = \frac{\#\text{ of carriers}}{m^2}$

Units of $\vec{K}$? A/m (current passing per unit length)

$d\vec{F} = (\vec{K} \times \vec{B})\,da$ similar derivation to before.

CQ: ribbon?

It is an experimental fact (observation) that the total charge is conserved.

This means you can pick any volume and,

Total inflow of charge = growth of net charge inside

Total outflow of charge = loss of net charge inside.

CQ: which states charge conservation?

Because $\vec{J}\cdot d\vec{a} = dI$ defines the total flow through an area $d\vec{a}$, the total outflow of some closed volume is, (bounded by A)

$$\oint_A \vec{J}\cdot d\vec{a}$$

the charge contained in the volume is,

$$\int_V \rho\, d\tau$$

The rate that charge is lost from the region is,

$$-\frac{d}{dt}\int_V \rho\, d\tau$$

so we can relate this to the total outflow,

$$\oint_A \vec{J}\cdot d\vec{A} = -\int_V \frac{d\rho}{dt}\, d\tau$$

using the divergence theorem,

$$\oint_A \vec{J}\cdot d\vec{A} = \int_V (\nabla\cdot\vec{J})\, d\tau = -\int_V \frac{d\rho}{dt}\, d\tau$$

$$\int_V \left(\nabla\cdot\vec{J} + \frac{d\rho}{dt}\right) d\tau = 0 \Rightarrow \nabla\cdot\vec{J} = -\frac{d\rho}{dt}$$

$$\nabla \cdot \vec{J} + \frac{d\rho}{dt} = 0$$ is the continuity equation.

This is the basic statement of charge conservation.

"outflow of current" + "increase in local charge" ~~must~~ must cancel!

<u>Full Summary</u>:

$\vec{J} = \rho \vec{v}$ = volume current density = Amps passing $a_\perp$

$\vec{K} = \sigma \vec{v}$ = surface current density = Amps passing $l_\perp$

$\vec{I} = \lambda \vec{v}$ = line current = Amps passing point

$\vec{J} = N_{vol}\, q \vec{v}$; $\vec{K} = N_{surf}\, q \vec{v}$; $\vec{I} = N_{line}\, q \vec{v}$