I still have questions about what we are trying to do with separation of variables in spherical coordinates.

A. Yes, definitely, let's talk about what we are trying to do (briefly).
B. I have some questions, but I think I got the gist of it. We can move on.
C. I got it, let's move on.

The ODE that describes the $R(r)$ part of our solution is:

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) = l(l+1)R$$

I claim this ODE gives rise to polynomial solutions.

Find a general solution for $R(r)$ in terms of l.

Let's take the Θ ODE term by term starting with $l = 0$

$$\frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) = 0$$

What are some possible solutions?

Hint: This is not as tricky as it might seem.

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

V everywhere on a spherical shell is a given constant, i.e. $V(R, \theta) = V_0$. There are no charges inside the sphere. Which terms do you expect to appear when finding V(inside)?

A. Many A_l terms (but no B_l's)
B. Many B_l terms (but no A_l's)
C. Just A_0
D. Just B_0
E. Something else!

The general solution for the electric potential in spherical coordinates with azimuthal symmetry (no ϕ dependence) is:

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Consider a metal sphere (constant potential in and on the sphere, remember). Which terms in the sum vanish outside the sphere? (Recall: $V \to 0$ as $r \to \infty$)

A. All the A_l's
B. All the A_l's except A_0
C. All the B_l's
D. All the B_l's except B_0
E. Something else

Given $V_0(\theta) = \sum_l C_l P_l(\cos\theta)$, we want to get to the integral:

$$\int_{-1}^{+1} P_l(u)\ P_m(u)\ du = \frac{2}{2+1}\ (\text{for } l = m)$$

we can do this by multiplying both sides by:

A. $P_m(\cos\theta)$
B. $P_m(\sin\theta)$
C. $P_m(\cos\theta)\sin\theta$
D. $P_m(\sin\theta)\cos\theta$
E. $P_m(\sin\theta)\sin\theta$