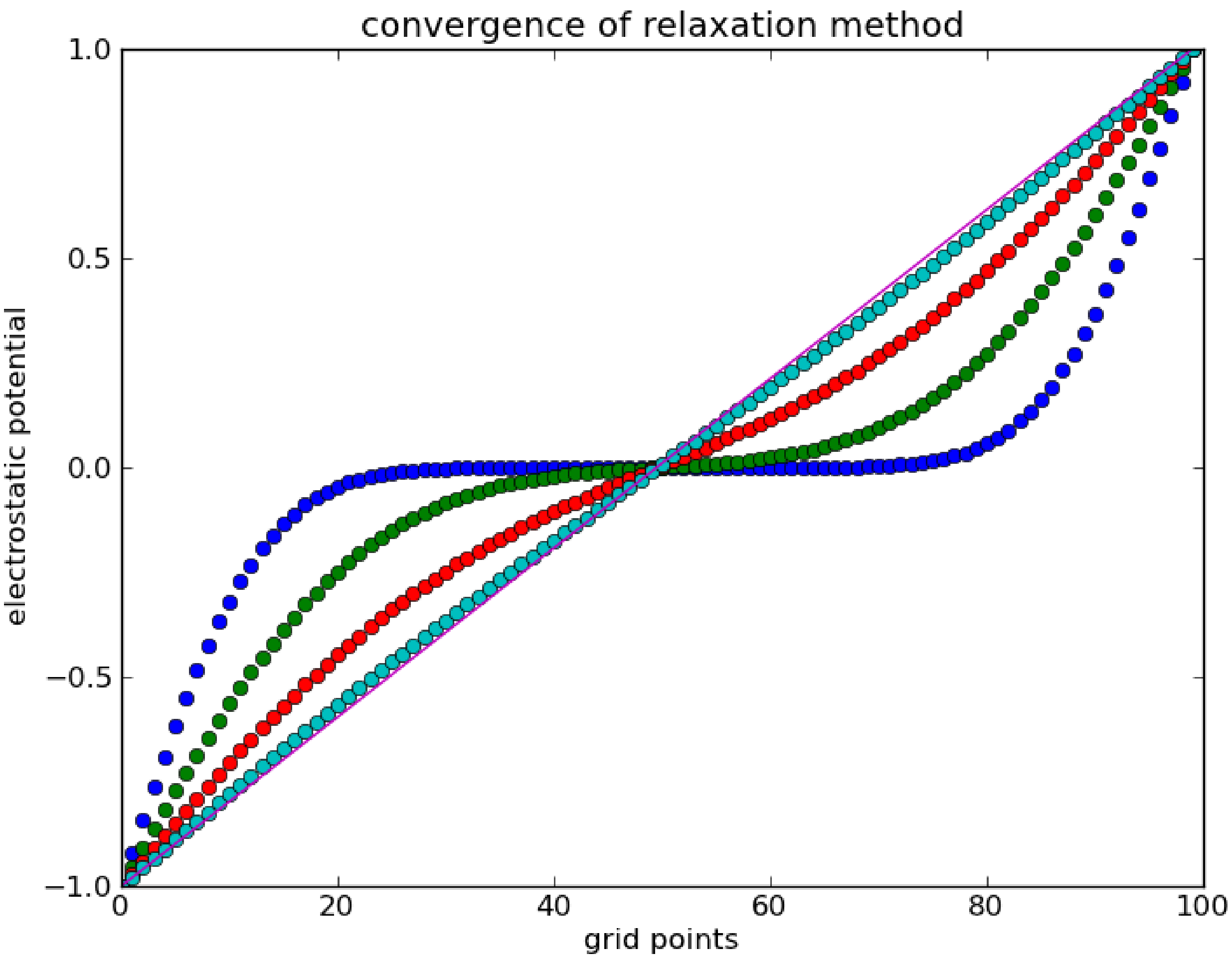
The solution to Laplace's equation in one Cartesian dimension is always linear. Why?

A. The general solution for $\frac{d^2V(x)}{dx^2} = 0$ is always a line.
B. The only way for a given point $V(x_i)$ (in 1d) to be the arithmetic average of its neighbors is for all point to be on a line.
C. Given boundary conditions, it's a unique solution.
D. None of these.
E. More than one of these.

METHOD OF RELAXATION



Consider a function $f(x)$ that is both continuous and continuously differentiable over some domain. Given a step size of a, which could be an approximate derivative of this function somewhere in that domain? $df/dx \approx$

A. $f(x_i + a) - f(x_i)$

B. $f(x_i) - f(x_i - a)$

C. $\frac{f(x_i+a)-f(x_i)}{a}$

D. $\frac{f(x_i)-f(x_i-a)}{a}$

E. More than one of these

If we choose to use:

$$\frac{df}{dx} \approx \frac{f(x_i + a) - f(x_i)}{a}$$

Where are we computing the approximate derivative?

A. a
B. x_i
C. $x_i + a$
D. Somewhere else

Taking the second derivative of $f(x)$ discretely is as simple as applying the discrete definition of the derivative,

$$f''(x_i) \approx \frac{f'(x_i + a/2) - f'(x_i - a/2)}{a}$$

Derive the second derivative in terms of f.

With the approximate form of Laplace's equation:

$$\frac{V(x_i + a) - 2V(x_i) + V(x_i - a)}{a} \approx 0$$

What is a the appropriate estimate of $V(x_i)$?

A. $1/2(V(x_i + a) - V(x_i - a))$
B. $1/2(V(x_i + a) + V(x_i - a))$
C. $a/2(V(x_i + a) - V(x_i - a))$
D. $a/2(V(x_i + a) + V(x_i - a))$
E. Something else

To investigate the convergence, we must compare the estimate of V before and after each calculation. For our `1D` relaxation code, V will be a `1D` array. For the kth estimate, we can compare V_k against its previous value by simply taking the difference.

Store this in a variable called `err`. What is the type for `err`?

A. A single number
B. A 1D array
C. A 2D array
D. ???

The Method of Relaxation also works for Poisson's equation (i.e., when there is charge!).

Given, $\nabla^2 V \approx \dfrac{V(x+a) - 2V(x) + V(x-a)}{a^2}$

Which equations describes the appropriate "averaging" that we must do:

A. $V(x) = \dfrac{1}{2}(V(x+a) - V(x-a))$

B. $V(x) = \dfrac{\rho(x)}{\varepsilon_0} + \dfrac{1}{2}(V(x+a) + V(x-a))$

C. $V(x) = \dfrac{a^2 \rho(x)}{2\varepsilon_0} + \dfrac{1}{2}(V(x+a) + V(x-a))$

SEPARATION OF VARIABLES (CARTESIAN)