We've analyzed a single current loop and determined both $\vec{A}$ + $\vec{B}$ for this situation. In material, we can model the atoms as little current loops with magnetic dipole moments. Now, the story is a bit more <u>subtle</u> and <u>interesting</u> than simply replacing each atom with a current loop as we will see soon. But for now let's think in terms of a bunch of little current loops.

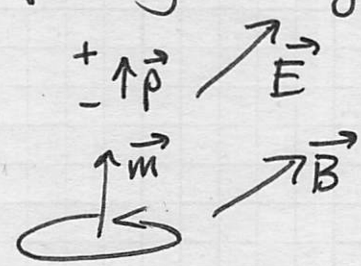
In that case we can talk about, <u>Magnetization</u>!

$$\frac{\text{magnetic dipole moment}}{\text{volume}} = \frac{\vec{m}}{V} = \vec{M}$$

(this is a lot like polarization $\vec{P} = \frac{\text{electric dipole moment}}{\text{volume}}$)

the analogy is pretty strong actually

for example,

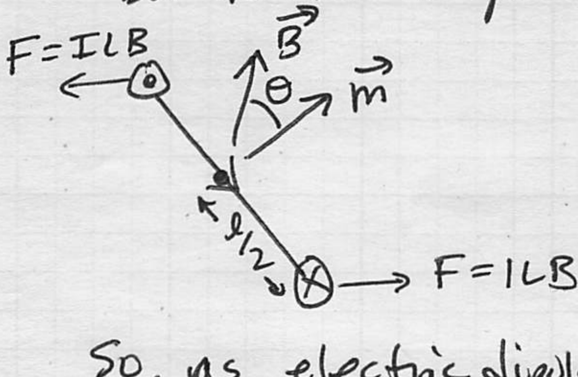


$\vec{\tau}_{\text{on E-dipole}} = \vec{p} \times \vec{E}$

$\vec{\tau}_{\text{on B-dipole}} = \vec{m} \times \vec{B}$

this torque result is easy to show with $\vec{F} = I d\vec{l} \times \vec{B}$.

But it's even simpler for a square loop

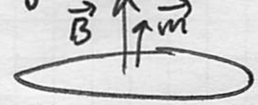


$\therefore\ \tau = \frac{l}{2} F\sin\theta$ on one wire

$\tau_{tot} = 2\frac{l}{2} IlB\sin\theta$

$= Il^2 B\sin\theta = mB\sin\theta$

So, as electric dipoles did, magnetic dipoles tend to line up with $\vec{B}$*



* sometimes not exactly or not parallel, but anti-parallel.

Also, the forces on dipoles are similarly analogous,

$$\vec{F}_{el} = \nabla(\vec{p}\cdot\vec{E}) \longleftrightarrow \vec{F}_{mag} = \nabla(\vec{m}\cdot\vec{B})$$

So, magnetic dipoles get "sucked" into regions of stronger magnetic field*
(* if they were already aligned by a torque.)

[You might be pretty confident that you can map a lot of your knowledge about $\vec{P}$ & $\vec{p}$ to $\vec{M}$ & $\vec{m}$, and that's true for a fair amount of things. But, there are also many differences (i.e. no monopoles, lots of dots become crosses, $\vec{B}$ is much more subtle, and $\vec{A}$ is much more complicated.
So Be careful!]

So we will begin to investigate how magnetic fields affect and are affected by matter.
It will be similar to electric fields...

- We apply $\vec{B}$, the matter adjusts → it magnetizes.
- You get a magnetization, $\vec{M}$, which creates $\vec{B}_{induced}$.
- The total magnetic field is different now.

There's a new complication now! ⟶ [$\vec{M}$ is not always in the direction of $\vec{B}$!]

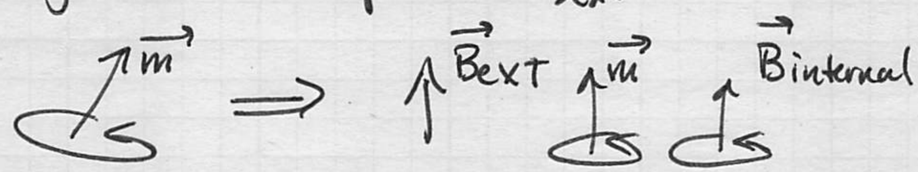
Before we begin to calculate anything related to $\vec{M}$ & $\vec{B}$, let's explore (from a classical atomic perspective) how matter can be affected by external magnetic fields.

There are two ways — one simpler than the other:

- Paramagnetism — $\vec{m}$ lines up with $\vec{B}_{ext}$ to produce a <u>parallel</u> $\vec{B}_{induced}$
- Diamagnetism — $\vec{m}$ points opposite $\vec{B}_{ext}$ and produces a <u>diametrically</u> opposed $\vec{B}_{int}$.

<u>Paramagnetism</u>

If the matter has little dipoles in it, we saw that they tend to line up with $\vec{B}_{ext}$.



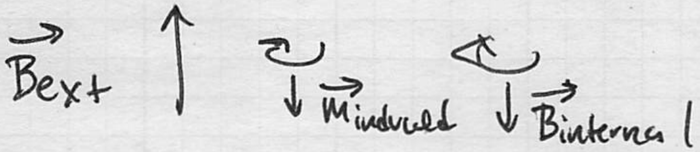
Thus, the total magnetic field <u>tends</u> <u>to get</u> <u>Stronger</u> $\rightarrow$ $\vec{B}_{int}$ lines up with $\vec{B}_{ext}$ to give $\vec{B}_{tot}$.

[Note! this is quite different from $\vec{E}$ where $\vec{p}$ lines up with $\vec{E}$, but the effect reduces E_{tot} b/c $\vec{E}_{induced}$ points opposite $\vec{E}_{ext}$ (monopole effect?) $\vec{p}\uparrow^{+}_{-}$ $^{+}_{-}\downarrow\vec{E}_{int}$ $\vec{E}_{ext}\uparrow$]

The atomic mechanism is the alignment of the electron spin (think spinning ball of charge) lining up with $\vec{B}_{ext}$. But! This phenomenon tends to be observed in materials with an odd # of electrons b/c electrons tend to pair up in opposing spin pairs $\Rightarrow$ Pauli Exclusion Principle.

Diamagnetism

The other thing that can happen to an atomic system is that the induced magnetic dipole moment is diametrically opposite the external field.



Here the internal magnetic field opposes the external field and thus the total field is reduced.

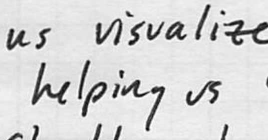
- The mechanism tends to be observed when paramagnetism is absent OR when an atom has an even number of electrons (because the opp. spins are paired off.) It's much weaker than paramagnetism.

How does diamagnetism arise? It seems a bit counter intuitive, doesn't it?

* The real answer to our question is that quantum mechanics gives rise to magnetic dipole moments that do interesting things in external fields.

Our Classical E&M explanation is "cheating", but it helps us to make sense of how this could arise and helps us visualize the mechanism. So it's really just helping us think about the phenomena, but we should not take it literally!

To start making sense of this, consider an electron in orbit with a constant velocity, v. at a radius, R.



* The "current" isn't steady, but with sufficiently fast it's an OK approx.

the electron travels $2\pi R$ in a time T such that,

$$vT = 2\pi R \qquad T = \frac{2\pi R}{v}$$

the "current" this electron produces is,

$$I = \frac{\text{charge}}{\text{time}} = \frac{e}{T} = \frac{eV}{2\pi R}$$

the magnetic dipole moment produced by this "current" is,

$$m = IA = \frac{eV}{2\pi R}\pi R^2 = \frac{evR}{2}$$

* We can observe where this magnetic dipole moment comes from (sort of) if we consider the angular momentum of the electron.

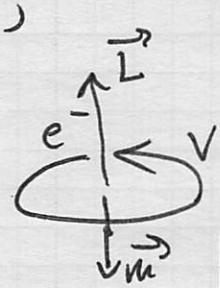
In a circular orbit, $L = m_e VR$ $\quad$ m_e is electron mass.

so the magnetic dipole moment is related to the angular momentum,

$$m = \frac{evR}{2} = \frac{e}{2m_e}(m_e VR) = \frac{eL}{2m_e} \quad \text{or}$$

$$\vec{m} = \frac{-e}{2m_e}\vec{L}$$

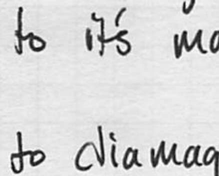
notice the minus sign! b/c electron has negative charge!



So the angular momentum of the electron contributes to it's magnetic dipole moment. (so does its spin).

Back to diamagnetism...

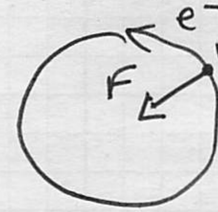
With the electron in this orbit, classically it's due to interaction with the nucleus, which for the sake of argument has charge, $+e$.



$$\vec{F}_{Elec} = \vec{F}_{net} \Rightarrow \frac{e^2}{4\pi\epsilon_0 R^2} = \frac{m_e V_0^2}{R}$$

* let V_0 be speed now

The net force in this situation is purely the electric force. $\frac{e^2}{4\pi\epsilon_0 R^2} = F$



Let's apply a magnetic field out of the page, which produces an additional force perpendicular to the velocity.

$$F_{new} = \frac{e^2}{4\pi\epsilon_0 R^2} + ev'B$$

In principle, this could change the speed, v', and the radius, R

In practice only the speed changes, v', and the radius, R, stays the same.

Big problem! We said $\vec{B}$ does no work and yet the electron changes it's speed, so there's a change in K! Who does this work?

⇒ As $\vec{B}$ is turned on, $d\vec{B}/dt \neq 0$, so there's an induced (circumferential) $\vec{E}$ field (Faraday's Law) that changes the kinetic energy of the electron.

⇒ That E-field is "just so" such that only $v \rightarrow v'$ and R stays the same.

So,

$$\frac{e^2}{4\pi\epsilon_0 R^2} + ev'B = \frac{m_e v'^2}{R}$$

But, $$\frac{e^2}{4\pi\epsilon_0 R^2} = \frac{m_e v_0^2}{R}$$

So that,

$$\frac{m_e v_0^2}{R} + ev'B = \frac{m_e v'^2}{R} \quad \text{or,}$$

$$\frac{m_e}{R}\left(v'^2 - v_0^2\right) = ev'B$$

For small B, we'd expect that v' isn't much different from v_0. so

$$(v'^2 - v_0^2) = (v' - v_0)(v' + v_0)$$

$$\simeq \delta v\,(2v_{avg}) \simeq \delta v\,(2v')$$

where δv is the change in the speed (can be + or −)

and $v_{avg} = \frac{v' + v_0}{2}$ or approximately v' (small change)

so,

$$\frac{m_e}{R}(\delta v)(2v') \cong ev'B$$ such that, the change is speed is,

$$\delta v \cong \frac{eBR}{2m_e}$$

★ How does this change in $v \to \delta v$ affect the magnetic dipole moment?

Recall that, $m = \frac{evR}{2}$ because on v is changing,

$$\delta m = \frac{eR}{2}\delta v = \frac{eR}{2}\left(\frac{eBR}{2m_e}\right) = \frac{e^2R^2}{4m_e}B$$

so if $\vec{B}_{ext}$ was in the +z direction, then the electron speeds up, thus m gets larger.

But! the magnetic dipole moment points opposite $\vec{B}$ (b/c electron) so,

$$\delta\vec{m} = -\frac{e^2R^2}{4m_e}\vec{B}$$

[add $\vec{B}$ and $\vec{m}$ changes in opposite way]

(If $\vec{B}$ was in −z direction, electron slows down, $|m|$ is smaller, but $\delta\vec{m}$ is up, still in $-\vec{B}$)

Bottom Line!

$\Rightarrow$ If a material has permanent dipole moments (For example, odd # of electrons) then paramagnetism dominates

$\Rightarrow$ If $\vec{m}_{atoms} = 0$ (e.g. even # of electrons), then it's likely to be diamagnetic.

Neither situation is permanent, the magnetization dissappears when $\vec{B}_{external}$ does!

Both of these effects are very small and not the "kitchen magnet" effect. $\rightarrow$ that's ferromagnetism!

If a material is paramagnetic, it is attracted into $\vec{B}_{ext}$.

If a material is diamagnetic, it repels from $\vec{B}_{ext}$.