Given the two diff. eq's :

$$\frac{1}{X}\frac{d^2X}{dx^2} = C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2} = C_2$$

where $C_1 + C_2 = 0$. Given the boundary conditions in the figure, which coordinate should be assigned to the negative constant (and thus the sinusoidal solutions)?

V1(y)

х





EXACT SOLUTIONS:

$$V(x, y) = \sum_{n=1}^{\infty} \frac{4V_0}{n\pi} \frac{1}{\cosh\left(\frac{n\pi}{2}\right)} \cosh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

APPROXIMATE SOLUTIONS:
(1 TERM; 20 TERMS)





Given that we want to solve Laplace's equation in 2D and that we have a description for the numerical second derivative of one variable,

$$f''(x) \approx \frac{f(x+a) - 2f(x) + f(x-a)}{a^2}$$

what is the appropriate numerical partial derivative for $V(x, y), \partial V/\partial x \approx$,

A.
$$[V(x + a) - 2V(x) + V(x - a)]/a^2$$

B. $[V(x + a, y) - 2V(x, y) + V(x - a, y)]/a^2$
C. $[V(y + a) - 2V(y) + V(y - a)]/a^2$
D. $[V(x, y + a) - 2V(x, y) + V(x, y - a)]/a^2$
E. More than one is correct

Given that the potential at any point is given by the average of the surrounding points,

$$V(x, y) \approx \frac{1}{4} [V(x + a, y) + V(x, y + a) + V(x - a, y) + V(x, y - a)]$$

Draft the psuedocode for finding the approximate potential.

SEPARATION OF VARIABLES (SPHERICAL)



Given $\nabla^2 V = 0$ in Cartesian coords, we separated V(x, y, z) = X(x)Y(y)Z(z). Will this approach work in spherical coordinates, i.e. can we separate $V(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$?

A. Sure.

B. Not quite - the angular components cannot be isolated, e.g., $f(r, \theta, \phi) = R(r)Y(\theta, \phi)$

C. It won't work at all because the spherical form of Laplace's Equation has cross terms in it (see the front cover of Griffiths)