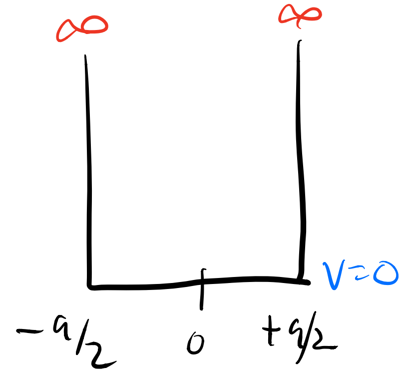
$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right]\psi_E(x)=E\psi_E(x) \Leftarrow$$

$$V(x)=\begin{cases}\infty & x<-a/2\\ 0 & -a/2<x<a/2\\ \infty & x>a/2\end{cases}$$



with $k^2 \equiv \frac{2mE}{\hbar^2} \Rightarrow \frac{d^2\psi}{dx^2}=-k^2\psi \Leftarrow$

general solutions: need two undetermined coeffs ← 2nd order

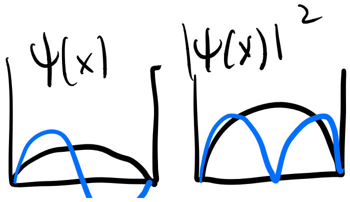
$\psi(x)=\underline{A}\cos kx+\underline{B}\sin kx$

$\psi(x)=\underline{C}\sin(kx+\underline{\delta})$ $\quad \psi(x)=\underline{D}\cos(kx+\underline{\xi})$

$\psi(x)=\underline{F}e^{ikx}+\underline{G}e^{-ikx}$ $\quad \psi(x)=\underline{H}e^{i(kx+\underline{\alpha})}$

All are valid but a clever choice makes all the difference.

$$\psi(x)=\underbrace{A\cos(kx)}_{\text{remains}}+\underbrace{B\sin(kx)}_{\text{vanish}}$$



$\psi(x)=\psi(-x)$ $\quad |\psi(x)|^2=|\psi(-x)|^2$

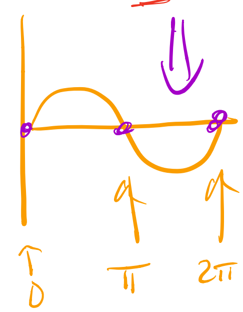
$$\psi(x) = C\sin(kx+\delta) \Longleftarrow$$

$$\left(\begin{aligned} \psi(x=a/2) &= 0 = C\sin\left(\tfrac{ka}{2}+\delta\right) \\ \psi(x=-a/2) &= 0 = C\sin\left(\tfrac{-ka}{2}+\delta\right) \end{aligned} \right\} \text{ B.C.}$$

No trivial solutions so, $C \neq 0$ ($\sin(\ ) = 0$)

$\frac{ka}{2} + \delta$ = (some integer) π

$-\frac{ka}{2} + \delta$ = (some other integer) π



① $\frac{ka}{2} + \delta = n\pi$ ② $-\frac{ka}{2} + \delta = m\pi$

① + ② $2\delta = \underbrace{(n+m)}_{\text{some integer}}\pi$

① − ② $ka = \underbrace{(n-m)}_{\text{some integer}}\pi$

$$\boxed{\begin{aligned} \delta &= \frac{(n+m)\pi}{2} \\ k &= \frac{(n-m)\pi}{a} \end{aligned}}$$

$m = 0 \rightarrow$ choosing energy scale

$\delta_n = n\pi/2$

$E_n \Longleftarrow \boxed{k_n = \frac{n\pi}{a}}$ quantization cond.

$$\psi(x) = C\sin\left(\frac{n\pi x}{a} + \frac{n\pi}{2}\right)$$

$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}} \Leftarrow k_n = \sqrt{\frac{2mE_n}{\hbar^2}}$$

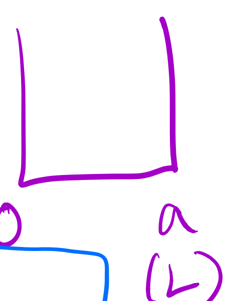
① no constraint ∞ sq. well

$n = 1, 2, \ldots . \longrightarrow$

(infinite # levels)

$$\langle x|\psi\rangle = \psi(x)$$

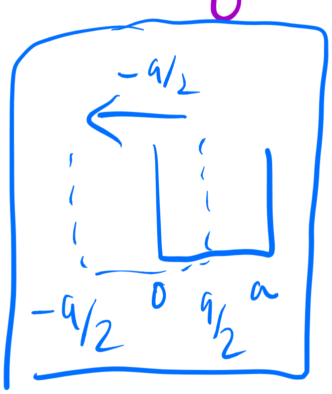
②



McIntyre

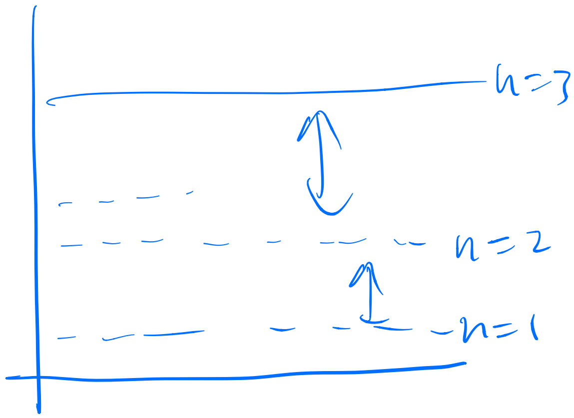
$$E_n = \frac{n^2\pi^2\hbar^2}{2m(\text{width})^2}$$

$\uparrow$ observable physics



$$\langle x|\psi_{mc}\rangle \propto \sin(\ )$$

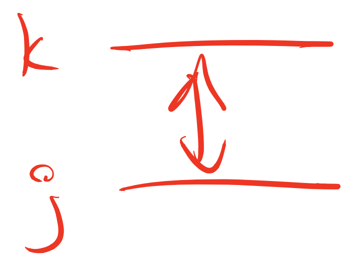
$$\langle x|\psi_{us}\rangle \propto \sin(\cdots + \delta)$$



$n, m \to \infty$

$k \quad j$

$$\Delta E = \frac{(k^2 - j^2)\pi^2\hbar^2}{2ma^2}$$



$k = j+1$

$$\Delta E = \frac{(\cancel{j^2} + 2j + 1 - \cancel{j^2})\pi^2\hbar^2}{2ma^2}$$

$$E = -\frac{13.6\,eV}{n^2}$$

$$-13.6\,eV\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$$

$$\Rightarrow \sqrt{E} \propto \sqrt{E-V_0}\,\tan(\sqrt{E})$$

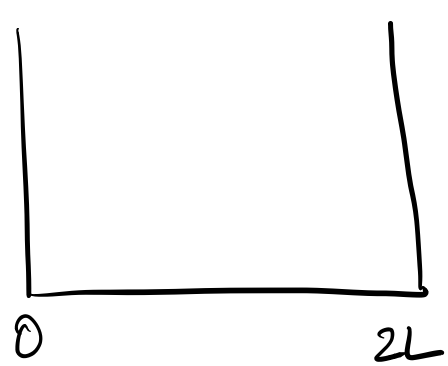
$\langle E_n|E_n\rangle = 1$ — normalization

$\langle E_m|E_n\rangle = 0$ — orthongonal

$$\langle x|\psi\rangle = \psi(x)$$

over what domain?

$$\psi(x) = A\cos\left(\frac{n\pi x}{L}\right) + B\sin\left(\frac{n\pi x}{L}\right)$$



$0 \to 2\pi$

$$\int_0^{2L} |\psi(x)|^2 dx = 1 \quad \text{normal}$$

$$\int_0^{2L} \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi x}{L}\right) dx = 0 \quad \text{orthog}$$

$Y_l^m(\theta, \phi)$ $R_{nl}(r)$