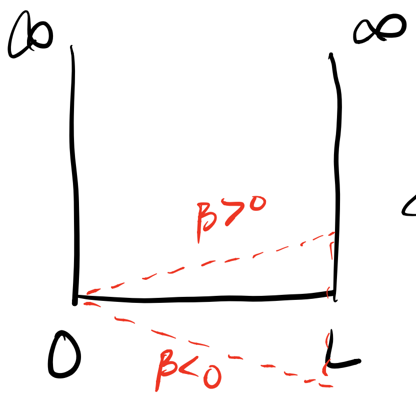
Perturb Infinite Square Well



$$H_0 = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

$$V(x) = \begin{cases} 0 & 0<x<L \\ \infty & x>L \\ & x<0 \end{cases}$$

$$E_n^{(0)} = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$|n^{(0)}\rangle \doteq \varphi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

$$H = H_0 + H' = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \beta x$$

$0<x<L$

$$\boxed{H' = \beta x}$$

$$E_n^{(1)} = \langle n^{(0)} | H' | n^{(0)} \rangle \Leftarrow$$

$$\int_0^L \varphi_n^{(0)*}(x)\,(\beta x)\,\varphi_n^{(0)}(x)\,dx$$

$H' = 0$ $x < 0$, $x > L$

$$= \frac{2}{L}\int_0^L \sin^2\left(\frac{n\pi x}{L}\right)\beta x\,dx = \frac{\beta L}{2}$$

$$E_n^{(1)} = \frac{\beta L}{2}$$

$$\boxed{E \simeq \frac{n^2\pi^2\hbar}{2mL^2} + \frac{\beta L}{2}}$$

$\beta > 0$

$\beta < 0$

$V(x) = \underline{\beta x} \leftarrow$ perturbation

$$|1\rangle = |1^{(0)}\rangle + |1^{(1)}\rangle + |1^{(2)}\rangle \text{ etc}$$

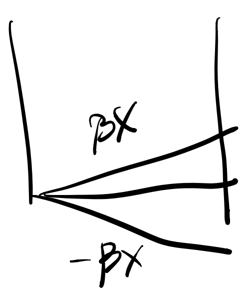
$$|1^{(1)}\rangle = \sum_{\underline{k \neq 1}} \frac{\underline{\langle 1^{(0)}|H'|k^{(0)}\rangle}}{E_1^{(0)} - E_k^{(0)}} |k^{(0)}\rangle$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$E_1^{(0)} - E_k^{(0)} \sim (1-k^2)$$

$n \to 1 \to \infty$

$$\text{denom} = \frac{1}{1-k^2} \Leftarrow \text{lower } \underline{\underline{\text{orders}}}$$



$$\langle 1^{(0)}| H' |k^{(0)}\rangle$$

$$= \int_0^L \sin\left(\frac{\pi x}{L}\right) \beta x \sin\left(\frac{k\pi x}{L}\right) dx$$

$$= \begin{cases} \neq 0 & k \text{ odd} \Leftarrow \\ = 0 & k \text{ even} \end{cases}$$

$$\sum_{k \neq 1} f(k) |k^{(0)}\rangle = |n^{(1)}\rangle$$

$k=3$ first contribution