Generalizing PT.

$$H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle \Leftarrow \text{unperturbed problem}$$

$$H_0+\lambda H'|n\rangle = E_n|n\rangle \Leftarrow \text{problem we want to solve}$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots$$

$$|n\rangle = |n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \lambda^2|n^{(2)}\rangle + \ldots$$

λ^0: $$H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle$$

λ^1: $$H_0|n^{(1)}\rangle + H'|n^{(0)}\rangle = E_n^{(0)}|n^{(1)}\rangle + E_n^{(1)}|n^{(0)}\rangle$$

λ^2: $$H_0|n^{(2)}\rangle + H'|n^{(1)}\rangle = E_n^{(0)}|n^{(2)}\rangle + E_n^{(1)}|n^{(1)}\rangle + E_n^{(2)}|n^{(0)}\rangle$$

etc.

$|n^{(i)}\rangle$

$\lambda^0: \quad (H_0 - E_n^{(0)})|n^{(0)}\rangle = 0$ original problem

$\lambda^1: \quad (H_0 - E_n^{(0)})|n^{(1)}\rangle = (E_n^{(1)} - H')|n^{(0)}\rangle$

$\lambda^2: \quad (H_0 - E_n^{(0)})|n^{(2)}\rangle = (E_n^{(1)} - H')|n^{(1)}\rangle + E_n^{(2)}|n^{(0)}\rangle$

First order corrections

$$\boxed{E_n^{(1)} = H'_{nn} = \langle n^{(0)}|H'|n^{(0)}\rangle}$$

diagonal H'

$$|n^{(1)}\rangle = \sum_{m \neq n} \underbrace{\frac{H'_{mn}}{(E_n^{(0)} - E_m^{(0)})}}_{C_m^{(1)}} |m^{(0)}\rangle$$

offdiag

$$\boxed{|n^{(1)}\rangle = \sum_{m \neq n} \frac{\langle m^{(0)}|H'|n^{(0)}\rangle}{(E_n^{(0)} - E_m^{(0)})} |m^{(0)}\rangle}$$

Second order correction

$$E_n^{(2)} = \sum_{m \neq n} \frac{|H'_{mn}|^2}{(E_n^{(0)} - E_m^{(0)})}$$ ← off diag

$$\boxed{E_n^{(2)} = \sum_{m \neq n} \frac{|\langle m^{(0)}|H'|n^{(0)}\rangle|^2}{(E_n^{(0)} - E_m^{(0)})}}$$

Notes:
- $H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle$ has to be able to solved
- Assumes each correction is smaller than H^0
  ⇒ preferable small contributions converge

- $E_n^{(0)} - E_m^{(0)} = 0$?

  won't work.

- For most classes,

  $|n^{(1)}\rangle \qquad E_n^{(1)} \text{ \& } E_n^{(2)}$

Example: $\vec{B}_0 = B_0 \hat{z} \qquad \vec{B}_2 = B_2 \hat{x}$

$$B_0 \gg B_2$$

$$H_0 \doteq \begin{pmatrix} \hbar\omega_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar\omega_0 \end{pmatrix}$$

$$E_+^{(0)} = \hbar\omega_0 \qquad E_0^{(0)} = 0 \qquad E_-^{(0)} = -\hbar\omega_0$$

$$H' \doteq \begin{pmatrix} 0 & \frac{\hbar\omega_2}{\sqrt{2}} & 0 \\ \frac{\hbar\omega}{\sqrt{2}} & 0 & \frac{\hbar\omega}{\sqrt{2}} \\ 0 & \frac{\hbar\omega_2}{\sqrt{2}} & 0 \end{pmatrix} = \omega_2 S_x$$

$$E_n^{(1)} = 0 \Rightarrow H'_{nn} = \langle n^{(0)} | H' | n^{(0)} \rangle$$