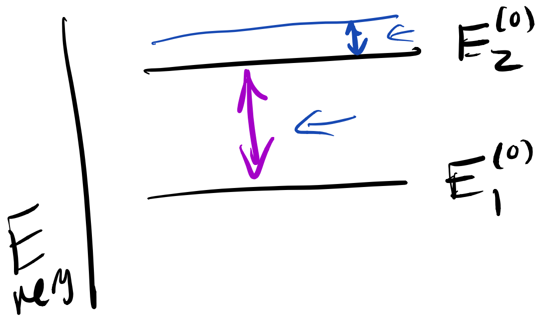
Time Independent Perturbation Theory



$$H_0 \doteq \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$H' \doteq \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

$$E_1 \simeq E_1^{(0)} + \lambda H'_{11} + \frac{\lambda^2 |H'_{12}|^2}{(E_2^{(0)} - E_1^{(0)})}$$

$$E_2 \simeq E_2^{(0)} + \lambda H'_{22} + \frac{\lambda^2 |H'_{21}|^2}{(E_1^{(0)} - E_2^{(0)})}$$

Spin 1 system

$$\vec{B}_0 = B_0 \hat{z} \Rightarrow \omega_0 \Rightarrow H_0$$

$$\vec{B}_1 = B_1 \hat{z} \Rightarrow \omega_1 \Rightarrow H'$$

$B_0 >> B_1$

$$H_0 = \omega_0 S_z = \begin{bmatrix} \hbar\omega_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar\omega_0 \end{bmatrix}$$

energy basis

$$H' = \omega_1 S_z = \begin{bmatrix} \hbar\omega_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar\omega_1 \end{bmatrix}$$

$E_+ = \hbar\omega_0$

$E_0 = 0$

$E_- = -\hbar\omega_0$

$E_n^{(1)} = \lambda H'_{nn} \Leftarrow$ generalizes

$$E_+^{(1)} = \hbar\omega_1$$

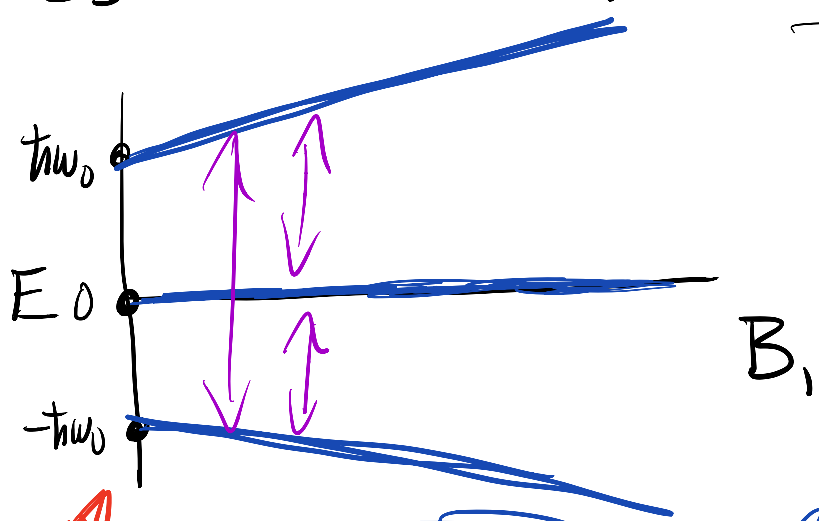
$$E_0^{(1)} = 0$$

$$E_-^{(0)} = -\hbar\omega_1$$

$$E_+ \simeq \hbar\omega_0 + \hbar\omega_1 = \hbar\underline{(\omega_0 + \omega_1)}$$

$$E_0 \simeq 0 + 0 = 0$$

$$E_- \simeq -\hbar\omega_0 - \hbar\omega_1 = -\hbar\underline{(\omega_0 + \omega_1)}$$



$$\omega_1 \propto B_1$$

$$\frac{gB}{2m}$$

Generalize P.T.

$$H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$$

$$(H_0 + H')|n\rangle = E_n |n\rangle$$

$$(H_0 + \lambda H')|n\rangle = E|n\rangle$$

Assumption: we can use power series

→ expect higher order terms to be vanishingly small

expect convergence.

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \lambda^3 E_n^{(3)} + \ldots$$

less overall contribution

$$|n\rangle = |n^{(0)}\rangle + \lambda |n^{(1)}\rangle + \lambda^2 |n^{(2)}\rangle + \lambda^3 |n^{(3)}\rangle + \ldots$$

$$\left(H_0+\lambda H'\right)\left(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\cdots\right)$$

$$=\left(E_n^{(0)}+\lambda E_n^{(1)}+\lambda^2 E_n^{(2)}+\cdots\right)$$

$$\left(|n^{(0)}\rangle+\lambda|n^{(1)}\rangle+\lambda^2|n^{(2)}\rangle+\cdots\right)$$

$$\lambda^0: \quad H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle$$

$$\lambda^1: \quad H_0|n^{(1)}\rangle + H'|n^{(0)}\rangle = E_n^{(1)}|n^{(0)}\rangle + E_n^{(0)}|n^{(1)}\rangle$$

$$\lambda^2: \quad H_0|n^{(2)}\rangle + H'|n^{(1)}\rangle = E_n^{(0)}|n^{(2)}\rangle + E_n^{(1)}|n^{(1)}\rangle + E_n^{(2)}|n^{(0)}\rangle$$

etc.