Time Ind. P.T.

⇒ First exposure Approx Methods in QM

⇒ General approx solution

$H_0|E_n\rangle = E_n|E_n\rangle$ ideal solution

zeroth order solution

↳ square well, QHO, basic spin sys., Hydrogen etc.

↳ Perturb → H'

$\boxed{\text{generally smaller effect, } H_0}$

slightly corrected $E, |n\rangle$

General Two Level Problem

zeroth

$$H_0 \doteq \begin{pmatrix} E_1^{(0)} & 0 \\ 0 & E_2^{(0)} \end{pmatrix}$$ $\Leftarrow$ energy basis

$$H_0 |1\rangle = E_1^{(0)} |1\rangle$$

$$H_0 |2\rangle = E_2^{(0)} |2\rangle$$

perturbation

$$H' \doteq \begin{pmatrix} H'_{11} & H'_{12} \\ H'_{21} & H'_{22} \end{pmatrix}$$

$$H|\psi\rangle = ? \qquad H = H_0 + H'$$

Introduce order parameter $\lambda \Rightarrow 1$

$\Rightarrow$ keep track of order of correction

$\lambda^0 \rightarrow$ zeroth

$\rightarrow$ $\lambda^1 \rightarrow$ first

$\rightarrow$ $\lambda^2 \rightarrow$ second etc.

$$H = H_0 + \lambda H' \doteq \begin{pmatrix} E_1^{(0)} + \lambda H'_{11} & \lambda H'_{12} \\ \lambda H'_{21} & E_2^{(0)} + \lambda H'_{22} \end{pmatrix}$$

$H'_{12} = H'^{\dagger}_{21}$ H has to be Hermitian

$H_0 \to E_n^{(0)}$ $H_0 + H' \to E_n = E$

$$H \doteq \begin{pmatrix} a & c \\ c^* & b \end{pmatrix}$$ form of matrix

Can we find exact solutions?

$$\det(H - IE) \doteq \det\begin{pmatrix} a-E & c \\ c^* & b-E \end{pmatrix} = 0$$

$$(a-E)(b-E) - cc^* = 0$$ find energy evalues.

$c^*c = |c|^2$

$$E^2 - E(a+b) + ab - |c|^2 = 0$$

$$\boxed{E = \frac{1}{2}(a+b) \pm \sqrt{\frac{1}{4}(a-b)^2 + |c|^2}}$$

Exact

Approximate Solution

H' contributions $\ll H_0$ contributions

$(b-a) \gg c$

$E_2^{(0)} \sim H_{22}$, $E_1^{(0)} \sim H_{11}$

$$\left(E_2^{(0)} + \lambda H_{22}\right) - \left(E_1^{(0)} + \lambda H_{11}\right)$$

H_{21}, H_{12}

$$E = \frac{1}{2}(a+b) \pm \frac{1}{2}(a-b)\left[1 + \frac{4|c|^2}{(a-b)^2}\right]^{1/2}$$

$$\boxed{E \simeq \frac{1}{2}(a+b) \pm \frac{1}{2}(a-b)\left[1 + \frac{2|c|^2}{(a-b)^2}\right]}$$

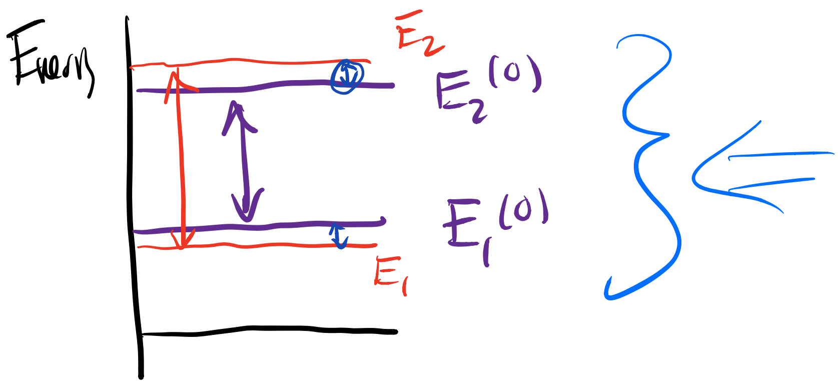
Approx Roots

$$E_1 \simeq a + \frac{|c|^2}{(a-b)}$$

$$E_2 \simeq b - \frac{|c|^2}{(a-b)}$$

Two level sys' Approx Energies

$E_2^{(0)}, E_1^{(0)}$ form the zeroth solution



$$E_1 \simeq a + \frac{|c|^2}{(a-b)} \qquad E_2 \simeq b - \frac{|c|^2}{(a-b)}$$

$$a = E_1^{(0)} + \lambda H'_{11} \qquad b = E_2^{(0)} + \lambda H'_{22}$$

$$c = \lambda H'_{12} \qquad c^* = \lambda H'_{21}$$

$$E_1 \simeq E_1^{(0)} + \lambda H'_{11} + \frac{\lambda^2 |H'_{12}|^2}{\left(E_1^{(0)} + \lambda H'_{11} - E_2^{(0)} - \lambda H'_{22}\right)}$$

$$E_2 \simeq E_2^{(0)} + \lambda H'_{22} + \frac{\lambda^2 |H'_{21}|^2}{\left(E_1^{(0)} + \lambda H'_{11} - E_2^{(0)} - \lambda H'_{22}\right)}$$

diag

$$E_n \simeq E_n^{(0)} + \underbrace{\lambda H'_{nn}}_{\text{first}} + \underbrace{\lambda^2 \frac{|H_{mn}|^2}{\left(E_n^{(0)} - E_m^{(0)}\right)}}_{\text{second}}$$

off diag

Two Level Result

$$E_n \simeq E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + \cdots$$

$$E_n^{(0)} \rightarrow H_{0,nn}$$

$$E_n^{(1)} \rightarrow H'_{nn}$$

$$E_n^{(2)} \rightarrow \frac{|H_{nm}|^2}{\left(E_n^{(0)} - E_m^{(0)}\right)}$$

doesn't work for deg. energies

Example: Spin 1/2 system

spin 1/2 $\quad \vec{B}_{tot} = \underbrace{B_0 \hat{z}}_{\text{we have solved}} + \underbrace{B_1 \hat{z} + B_2 \hat{x}}_{H' \text{ perturbation}}$

Assume: $B_0 >> B_1 \,\&\, B_2$

$$H_0 = \omega_0 S_z \quad \text{) original}$$

$$H' = \omega_1 S_z + \omega_2 S_x \quad \text{) perturbation}$$

$$\underline{H_0} = \omega_0 S_z \doteq \frac{\hbar}{2}\begin{pmatrix} \omega_0 & 0 \\ 0 & \underline{-\omega_0} \end{pmatrix} \Leftarrow$$

S_z basis.

energy basis $|+\rangle_z$ $|-\rangle_z$

$$H' = \omega_1 S_z + \omega_2 S_x$$

$$\doteq \underline{\frac{\hbar}{2}}\begin{pmatrix} \underline{\omega_1} & \omega_2 \\ \omega_2 & \underline{-\omega_1} \end{pmatrix}$$

$$H_0 + H' \doteq \frac{\hbar}{2}\begin{pmatrix} \omega_0 + \omega_1 & \omega_2 \\ \omega_2 & \omega_0 - \omega_1 \end{pmatrix}$$

$$E_+ \simeq E_+^{(0)} + \lambda H'_{11} + \lambda^2 \frac{|H_{12}|^2}{\left(E_+^{(0)} - E_-^{(0)}\right)}$$

just the prescription

$$E_+ \simeq \frac{\hbar}{2}\omega_0 + \lambda\frac{\hbar}{2}\omega_1 + \lambda^2 \frac{\left(\frac{\hbar}{2}\omega_2\right)^2}{(\hbar\omega_0)}$$

$\lambda = 1$

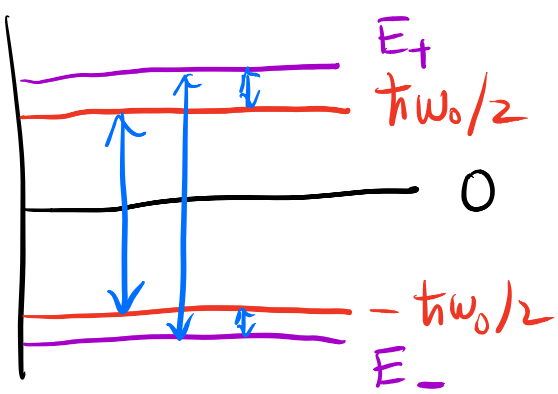
$$E_+ \simeq \frac{\hbar}{2}\omega_0 + \underline{\frac{\hbar}{2}\omega_1} + \underline{\frac{\hbar}{4}\frac{\omega_2^2}{\omega_0}}$$

$$\omega_0 \gg \omega_1$$

$$\omega_0 \gg \omega_2$$

$$E_- = -\frac{\hbar}{2}\omega_0 + \lambda\left(-\frac{\hbar}{2}\omega_1\right) + \lambda^2 \frac{\left(\frac{\hbar}{2}\omega_2\right)^2}{(-\hbar\omega_0)}$$

$$E_- = -\left(\frac{\hbar}{2}\omega_0 + \frac{\hbar}{2}\omega_1 + \frac{\hbar}{4}\frac{\omega_2^2}{\omega_0}\right)$$



$B_0 \gg B_1 \;\&\; B_2$