

# Schrodinger Time Evol.

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

time evolution      energy as QM system

eigenstates of  $H(t)$

$$H(t) |E_n\rangle = E_n |E_n\rangle$$

↑ states      ↑ values

$$H(t) = H \quad \text{no time dep.}$$

$$|\psi\rangle = \sum_n c_n |E_n\rangle$$

expansion in energy basis

in general

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

$$\langle E_m | E_n \rangle = \underline{\underline{\delta_{mn}}}$$

Solve Energy Eigenvalue

$$\hat{H} = \begin{pmatrix} 2 & 3 & 2 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \end{pmatrix}$$

$$\det(\hat{H} - I\lambda) = 0$$

$$\lambda = \sum \begin{matrix} \text{energy} \\ \text{eigenvalues} \end{matrix}$$

Activity

$$H = \hbar\omega_0 \begin{pmatrix} 2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 2 \end{pmatrix} |a\rangle, |b\rangle, |c\rangle$$

a) not diagonal  $\rightarrow$  not in basis

b)  $\lambda = \hbar\omega_0$      $H_{2i} = H_{i2} = 0$   
 $\uparrow \leftarrow \langle 2 | H | i \rangle = 0$


$$|E_{hw_0}\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \leftarrow$$

c) explicit calc.

$$d) \lambda = \begin{cases} E + \Delta E \\ E - \Delta E \end{cases}$$

$$\det(H - \lambda) = 0$$

(1- $\lambda$ ) quadratic in  $\lambda$   
roots



$$\underline{E = 2} \quad \underline{\Delta E = \pm 1/2}$$

$$|E_{3/2 hw_0}\rangle \doteq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \text{ antisym}$$

$$|E_{5/2 hw_0}\rangle \doteq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ sym}$$

$$\lambda = \begin{cases} \hbar\omega_0 & \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \\ 3/2\hbar\omega_0 & \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix} \\ 5/2\hbar\omega_0 & \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} \end{cases}$$


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$$H = \begin{bmatrix} a & 0 \\ 0 & b & c \end{bmatrix}$$

$$H = \begin{bmatrix} a & 0 & d \\ 0 & b & 0 \\ d & 0 & a \end{bmatrix}$$

$$H = \begin{bmatrix} a & e & d \\ e & b & e \\ d & e & a \end{bmatrix}$$


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Value of energy basis

⇒ spectrum observe in exp.  
lab, astrophysical obs,  
collision, etc.

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$$

time evol.
energy operator

Postulate 6.

$$H(t) = H$$

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

$$\begin{aligned}
 |\psi(t)\rangle &= c_1 e^{-i\omega_0 t} |E_1\rangle \\
 &+ c_2 e^{-i\frac{3}{2}\omega_0 t} |E_2\rangle \\
 &+ c_3 e^{-i\frac{5}{2}\omega_0 t} |E_3\rangle
 \end{aligned}$$

$$|E_1\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |E_2\rangle \doteq \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$$

$$|E_3\rangle \doteq \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

Probability measure  $E_3$ ?

$$|\langle E_3 | \Psi(t) \rangle|^2 = |c_3|^2$$

$$\langle E_m | E_n \rangle$$



Energy states  
are stationary  
states.