$$\boxed{\begin{aligned} H|nlm\rangle &= \frac{-c}{n^2}|nlm\rangle \qquad c = 13.6\,\text{eV Hydrogen} \\ L^2|nlm\rangle &= l(l+1)\hbar^2|nlm\rangle \\ L_z|nlm\rangle &= m\hbar\,|nlm\rangle \end{aligned}}$$

$$|nlm\rangle \doteq \psi_{nlm}(r,\theta,\phi) = R_{nl}(r)\,Y_l^m(\theta,\phi)$$

Time Evol

$$|\psi(t)\rangle \doteq \psi_{nlm}(r,\theta,\phi,t)$$

$$= R_{nl}(r)\,Y_l^m(\theta,\phi)\,e^{-iE_n t/\hbar}$$

$$E_n = -\frac{c}{n^2}$$

Superposition

$$|\Psi(t)\rangle = \sum_{n,l,m} C_{nlm} R_{nl}(r) Y_l^m(\theta,\phi) e^{-iE_n t/\hbar}$$

$$\langle nlm|\Psi(t)\rangle = C_{nlm}$$

Example: 1s 2p $(m=0)$

$$\Psi(t) \rightarrow P(t) \propto \cos\left(\frac{E_2-E_1}{\hbar}t\right)$$

oscillatory dipole

Example: 2s 2p $(m=0)$

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}|200\rangle + \frac{1}{\sqrt{2}}|210\rangle$$

$$\Psi(r,\theta,\phi,t) = \frac{1}{\sqrt{2}}\psi_{200}e^{-iE_2 t/\hbar} + \frac{1}{\sqrt{2}}\psi_{210}e^{-iE_2 t/\hbar}$$

$$= \frac{1}{\sqrt{2}}\left(\psi_{200}(r,\theta,\phi) + \psi_{210}(r,\theta,\phi)\right)e^{-iE_2 t/\hbar}$$

$$\psi(r,\theta,\phi,t) = \frac{1}{\sqrt{2}}\left(\psi_{200}+\psi_{210}\right) e^{-iE_2 t/\hbar}$$

$$P(r,\theta,\phi,t) = |\psi|^2$$

$$= \frac{1}{2}\left(\psi_{200}^* + \psi_{210}^*\right)\left(\psi_{200}+\psi_{210}\right)$$

$$\times \underbrace{\left(e^{+iE_2 t/\hbar}\right)\left(e^{-iE_2 t/\hbar}\right)}_{=1}$$

$$P(r,\theta,\phi,t) = P(r,\theta,\phi)$$

model fixed dipole

Superposition States (normalized)

→ E degenerate

n same l, m diff ✓

$$\frac{1}{\sqrt{2}}|200\rangle + \frac{1}{\sqrt{2}}|211\rangle$$

→ L^2 degeneracy?

l same $\quad$ n, m diff ✓

$$\frac{1}{\sqrt{2}}|210\rangle + \frac{1}{\sqrt{2}}|31,-1\rangle$$

$l < n$ $\quad$ $n = 1, \ldots, \infty$

$l = 0, \ldots, n-1$

→ L_z degeneracy?

m same $\quad$ n, l diff ✓

$$\frac{1}{\sqrt{2}}|210\rangle + \frac{1}{\sqrt{2}}|320\rangle$$

$$\langle H\rangle = \frac{-c}{9} = \frac{-c}{n^2} \quad \langle L_z\rangle = 0$$

$n=3$ $\quad$ $m=0$

$L^2 = 6\hbar^2$ or $2\hbar^2$

$(l = 2 \ \& \ 1)$

$$\frac{1}{\sqrt{2}}\left(|320\rangle + |310\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|321\rangle + |31-1\rangle\right)$$

$$\langle L_z\rangle = \Sigma P_{L_z}(\text{e-value})$$

$c/9$ $c/4$ $n = 3, 2$

$$\langle L^2\rangle = 2\hbar^2 \quad l = 1$$

$m = 1, 0$