$$|\psi\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{i}{\sqrt{6}}|00\rangle$$

$|lm\rangle \Leftarrow$ particle on a sphere

L_z:

(1) $2\hbar$: 0

$-\hbar$: $1/2$

$0\hbar$: $\underline{1/2} \Leftarrow$ deg $|10\rangle$ & $|00\rangle$

$$\sum_{l \text{ contain } m} |\langle lm|\psi\rangle|^2$$

(2) $\langle L_z\rangle = \sum(\text{Prob})(\text{eigen}) = \frac{1}{2}(-\hbar) + \frac{1}{2}(0)$

$= -\hbar/2$

degeneracy $\rightarrow$ any operator real eigenvalue

$L_z|lm\rangle = m\hbar|lm\rangle$

any share $m \rightarrow$ deg. L_z

L^2:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{i}{\sqrt{6}}|00\rangle$$

$$L^2|lm\rangle = l(l+1)\hbar^2|lm\rangle$$

Prob $\nearrow$ $2\hbar^2$, $l=1$, $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

Prob $\searrow$ $0\hbar^2$, $l=0$, $\frac{1}{6}$

$$\langle L^2\rangle = \sum \text{Prob}\,(\text{eigen}) = \frac{5}{6}(2\hbar^2) + \frac{1}{6}(0)$$

$$= \frac{10}{6}\hbar^2 = \frac{5}{3}\hbar^2$$

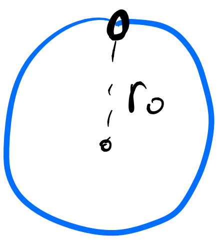
4

$\langle H\rangle$: $l=1$,

$$H|lm\rangle = \frac{\hbar^2}{2I}l(l+1)|lm\rangle$$

$$H = \frac{L^2}{2I}$$ particle on sphere

$$\frac{m^2\hbar^2}{2I} \; ? \quad \leftarrow \quad H \Leftrightarrow L_z$$



$$H_{ring} = \frac{L_z^2}{2I}$$

$$H_{ring}|lm\rangle = \frac{m^2\hbar^2}{2I}|lm\rangle$$

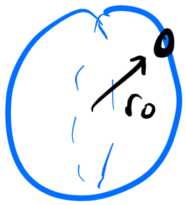
$$|\psi\rangle = a|l_1 m\rangle + b|l_2 m\rangle$$

degenerate L_z, H particle on ring

not degen. L^2

$$L^2|lm\rangle = l(l+1)\hbar^2|lm\rangle$$

any l can have $m = -l, \dots, l$



$$H_{sphere} = \frac{L^2}{2I}$$

$$H_{sphere}|lm\rangle = \frac{L^2}{2I}|lm\rangle$$

$$= \frac{l(l+1)\hbar^2}{2I}|lm\rangle$$

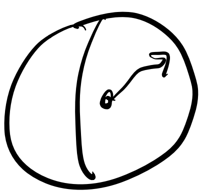
$$|\psi\rangle = a|l_1 m\rangle + b|l_2 m\rangle$$

not deg. for H, L^2 — particle on sphere

deg. for L_z — share m.

Both Cases.

$$|\psi\rangle = a|l_1 m_1\rangle + b|l_2 m_1\rangle + c|l_2 m_2\rangle + \dots$$



$$\psi(r,\theta,\phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$$

$$H_{3D}|nlm\rangle = \frac{-E_{tot}}{n^2}|nlm\rangle$$

electronic $\rightarrow R$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|1,-1\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{i}{\sqrt{6}}|00\rangle$$

$|lm\rangle$

$\langle L_x \rangle = ?$

$L_y|\psi\rangle$

Method 1

$L_y|1,-1\rangle$

$L_y|10\rangle$

$L_y|00\rangle$

Method 2

$|{}_y\langle 1,1|\psi\rangle|^2$

$|{}_y\langle 10|\psi\rangle|^2$

$\vdots$

$|{}_y\langle 1-1|\psi\rangle|^2$

spin 1

Method 3

$|\psi\rangle \doteq \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$l=1$

$\boxed{|\psi\rangle \quad \text{state vectors}}$

$\langle x|\psi\rangle$ or $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ representation state vector

$\downarrow$

$\psi(x,y,z)$

$\psi(r,\theta,\phi)$

spin	1/2	2D
spin	1	3D
spin	3/2	4D
$\vdots$		