Day 1 & 2

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$ $\swarrow S_z$

add & subtract

$$|-\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

1. $$|+\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_x + |-\rangle_x\right)$$ $\swarrow S_x$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_x - |-\rangle_x\right)$$

linear combination

eigenstates not unit vectors unless their in their own basis. ⟸

2. S_x basis

$$|+\rangle_x \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|-\rangle_x \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boxed{\begin{aligned} &(1)\quad S_z|+\rangle = \frac{\hbar}{2}|+\rangle \\ &(2)\quad S_z|-\rangle = -\frac{\hbar}{2}|-\rangle \end{aligned}}$$

$$S_z \doteq \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Determine operator in S_x

$$(1)\quad \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$(2)\quad \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}}_{S_x \text{ basis}} = \underbrace{-\frac{\hbar}{2}\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}}_{S_y \text{ basis}}$$

$\Big\}$ a, b, c, d

$$S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Determining operator

$$A \doteq \begin{pmatrix} \langle + | A | + \rangle & \boxed{\langle + | A | - \rangle} \\ \langle - | A | + \rangle & \langle - | A | - \rangle \end{pmatrix}$$

S_z basis

$$A_x \doteq \begin{pmatrix} {}_x\langle +|A|+\rangle_x & {}_x\langle +|A|-\rangle_x \\ {}_x\langle -|A|+\rangle_x & {}_x\langle -|A|-\rangle_x \end{pmatrix}$$

$$S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \longrightarrow \boxed{\lambda = \pm\frac{\hbar}{2}}$$

2x2

$$\det|S_z - I\lambda) = 0$$

quadratic

Spin 1 $\rightarrow$ 3x3 $\longrightarrow$ cubic

$\lambda_1, \lambda_2, \lambda_3$

$$\lambda_+ = +\frac{\hbar}{2} \qquad \lambda_- = -\frac{\hbar}{2}$$

$$|+\rangle \doteq \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad |-\rangle \doteq \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} \qquad ? \qquad \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$S_z|+\rangle = +\frac{\hbar}{2}|+\rangle$$ e-value eqn.

$$\cancel{\frac{\hbar}{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \cancel{\frac{\hbar}{2}}\begin{pmatrix} a \\ b \end{pmatrix}$$

$0a + b = a \qquad b = a$

$1a + 0b = b \qquad a = b$

free to choose

$\langle +|+\rangle = 1 \qquad |a|^2 + |b|^2 = 1$

$a = 1/\sqrt{2} = b$

Overall phase doesn't matter

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \frac{1}{2}\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$\times e^{i\pi}$

$\langle\psi|\psi\rangle$

$\underbrace{(e^{-i\pi})(e^{i\pi})}_{1}$

$$|\psi\rangle = \frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle$$

proj. along S_x e-states?

$$\text{Proj}_{x+} = \underline{|+\rangle_x}\ {}_x\langle + | \underbrace{\psi\rangle}$$

dot

$|+\rangle_x$, $|-\rangle_x$

basis

$\vec{a} \cdot \vec{b}$ = proj$_{a \text{ along } b}$ (assumption carry b direction)

$$\text{Proj}_{x+} = \frac{7}{5\sqrt{2}}|+\rangle_x$$

$$\text{Proj}_{x-} = \frac{-1}{5\sqrt{2}}|-\rangle_x$$

$$\text{Prob}_{+x} = \left| {}_x\langle + | \text{Proj}_{x+}\rangle \right|^2$$

$$= \frac{49}{50}$$

$$Prob\_x = 1/50$$