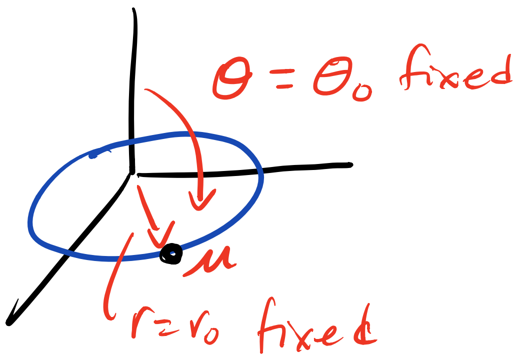
Particle on a Ring



Only free parameter ϕ

1D problem

B/c $\theta = \theta_0$, $r = r_0$

$$\boxed{-\frac{\hbar^2}{2\mu r_0^2}\frac{d^2\psi}{d\phi^2} + V(r_0)\,\psi = E\psi}$$

choose $V(r_0) = 0$

$$\psi(r,\theta,\phi) = \psi(r_0,\theta_0,\phi) = \Phi(\phi)$$

$$-\frac{\hbar^2}{2\mu r_0^2}\frac{d^2\Phi}{d\phi^2} = E\Phi$$

$I \equiv \mu r_0^2$

$$\boxed{\frac{-\hbar^2}{2I}\frac{d^2\Phi}{d\phi^2} = E\Phi}$$

particle on a ring.

$\Downarrow$

operator eqn. $\Rightarrow$ $\underline{\hat{O}}\Phi = E\Phi$

$$L_z = xp_y - yp_x$$

$$L_z(x,y,z) \rightarrow L_z(r,\theta,\phi)$$

$$\frac{\partial}{\partial x} = \underbrace{\frac{\partial r}{\partial x}\frac{\partial}{\partial r}}_{\substack{0 \\ r=r_0}} + \underbrace{\frac{\partial \theta}{\partial x}\frac{\partial}{\partial \theta}}_{\substack{0 \\ \theta=\theta_0}} + \frac{\partial \phi}{\partial x}\frac{\partial}{\partial \phi}$$

$$\frac{d\phi}{dx} \qquad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = r_0 \sin\theta \cos\phi$$
$$y = r_0 \sin\theta \sin\phi$$

$$L_z \propto \frac{\partial}{\partial\phi} \qquad L_z^2 \propto \frac{\partial^2}{\partial\phi^2}$$

$$-\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\phi^2}\Phi = E\Phi$$

$$\propto L_z^2 \quad \rightarrow L_z^2|lm\rangle = m^2\hbar^2|lm\rangle$$