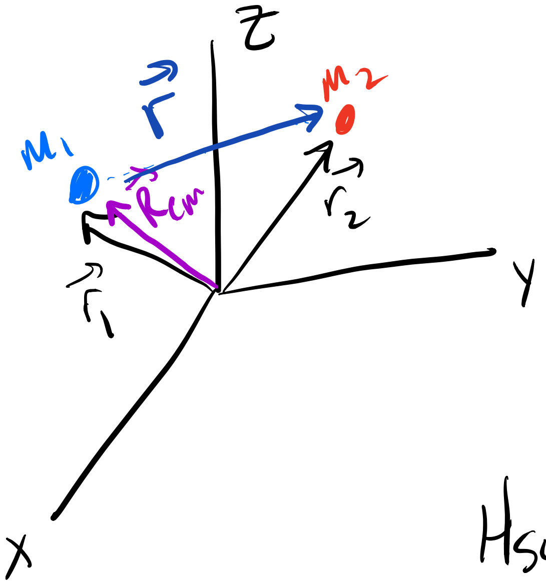


$$H_{sys} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(\vec{r}_1, \vec{r}_2)$$

central potential

$$V(\vec{r}_1, \vec{r}_2) = V(|\vec{r}_2 - \vec{r}_1|)$$

$$H_{sys} = \underbrace{\frac{|P_{tot}|^2}{2M}}_{CM} + \underbrace{\frac{|P_{rel}|^2}{2\mu}} + V(r)$$

$$R_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$m_1 << m_2$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

nucleus & electron

1 MeV · 511 keV

$$H_{sys} = H_{cm} + H_{rel}$$

$\rightarrow E_{rel}$ electron

$$H_{relative} = \frac{|P_{rel}|^2}{2\mu} + V(r)$$

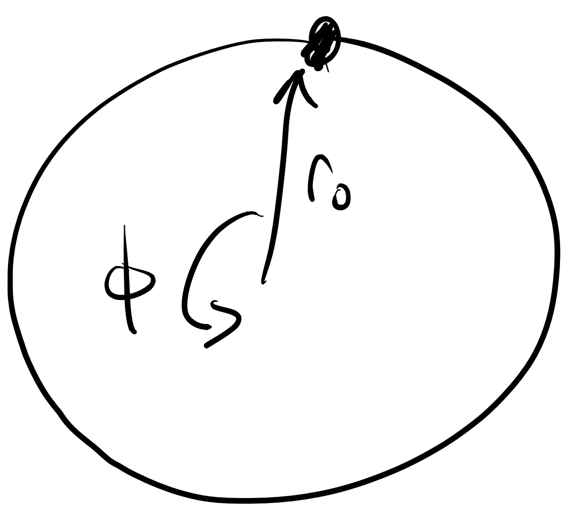
Spherical

$$H_{nel} \, | E_{nel} \rangle = E_{nel} \, | E_{nel} \rangle$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$

Laplacian in spherical

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right)\psi(r,\theta,\phi) = E\,\psi(r,\theta,\phi)$$



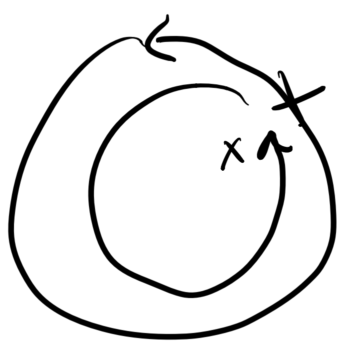
particle on a ring

Debrief

fix r_0, θ_0

$$\frac{d^2\psi}{d\phi^2} = -\underbrace{\left(\frac{2r_0^2 \mu (E-V_0)}{\hbar^2}\right)}_{\text{Constant} > 0}\psi$$

$V_0 \to 0$



$$\frac{d^2\psi}{d\phi^2} = -B\psi \qquad B>0$$

$$\psi = C e^{i\sqrt{B}\phi} + D e^{-i\sqrt{B}\phi}$$

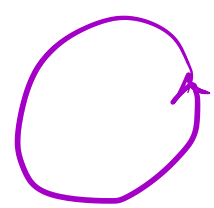
2π periodicity $\rightarrow$ $\sqrt{B} = m = 0, \pm 1, \pm 2 \ldots$

$m = 0, \pm 1, \pm 2, \ldots$

$$\boxed{\psi(\phi) = C e^{im\phi}}$$

$$\underline{\psi(\phi) = \psi(\phi + 2n\pi)}$$

$$\langle \psi | \psi \rangle = \int_0^{2\pi} \psi^* \psi \, d\phi$$



$$= |C|^2 2\pi = 1$$

$$\boxed{\psi = \frac{1}{\sqrt{2\pi}} e^{im\phi}}$$